Block 07 — Power-relevant Figures

Student Group

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Learning objectives

- Define and compute **input/output power**, **losses**, **efficiency** η and **utilization rate** ε for DC sources and loads.
- Use the **real source model** with internal resistance $R_{\rm i}$ to compute operating point $(U_{\rm L}, I_{\rm L})$, $P_{\rm L}$ and $P_{\rm loss}$.
- Understand the different design goals:
    1. **High efficiency** (power engineering): $R_{\rm L} \gg R_{\rm i}$.
    2. **Maximum power transfer** (communications): $R_{\rm L} = R_{\rm i}$.
- Combine efficiencies along a **power-flow chain**.
- Relate these figures to **Thevenin/Norton** equivalents and the **loaded voltage divider**.

90-minute plan

1. Warm-up (8 min): recall passive/active sign convention; quick unit check for $P=U\cdot I$.
2. Core concepts (35 min): real source model; definitions of η and ε; design goals; chain efficiency.
3. Worked example (10 min): battery + internal resistance + load.
4. Two-port view & loaded divider (12 min): quick Thevenin/Norton recap; loaded divider formulas.
5. Practice (20 min): 3 short exercises (see panels below).
6. Wrap-up (5 min): summary + pitfalls.

Conceptual overview

1. Real sources are modeled by an **ideal source** plus **internal resistance** $R_{\rm i}$; the terminal voltage **drops under load**.
2. **Efficiency** η compares *delivered* to *drawn* power. In the simple DC source–load case, $\displaystyle \eta=\frac{R_{\rm L}}{R_{\rm L}+R_{\rm i}}$ (dimensionless). High-efficiency design wants $R_{\rm L}\gg R_{\rm i}$.
3. **Utilization rate** ε compares delivered power to the **maximum** available from the ideal source: $\displaystyle \varepsilon=\frac{R_{\rm L}R_{\rm i}}{(R_{\rm L}+R_{\rm i})^2}$. It peaks at $R_{\rm L}=R_{\rm i}$ with $\varepsilon_{\max}=25~\%$. This is the **maximum power transfer** condition.
4. Different goals → different $R_{\rm L}$:
    - **Power engineering**: maximize η → $R_{\rm L}\gg R_{\rm i}$.
    - **Communications** (matching, antennas, RF): maximize $P_{\rm L}$ → $R_{\rm L}=R_{\rm i}$, $\eta=50~\%$.

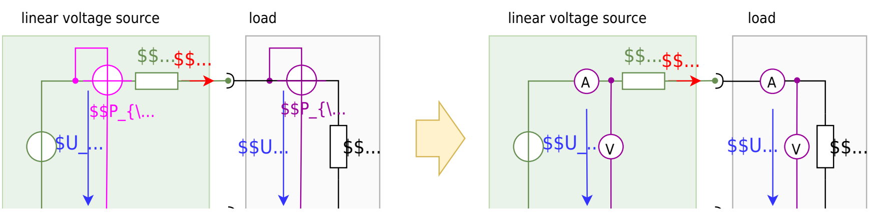
Core content

Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In figure 1 the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source $P_\rm S$ and the input power of the load $P_\rm L$.

Fig. 1: Power measurement on linear voltage source



Power and Characteristics in Diagrams

The simulation in figure 2 shows the following:

- The circuit with linear voltage source (U_0 and $R_\rm i$), and a resistive load $R_\rm L$.
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor $R_\rm S$ (English: shunt) and a voltage measurement for $U_\rm S$. The power is then: $P_\rm L = {{1}\over{R_\rm S}}\cdot U_\rm S \cdot U_\rm L$.
- in the oscilloscope section (below).

  - On the left is the power $P_\rm L$ plotted against time in a graph.
  - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance $R_\rm L$, with which the value of the load resistance $R_\rm L$ can be changed.

Now try to vary the value of the load resistance $R_\rm L$ (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 2: power adjustment

figure 3 shows three diagrams:

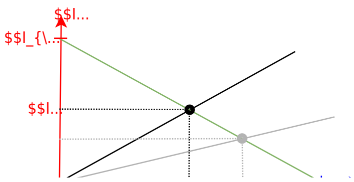
- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power $P_\rm S$ and consumer power $P_\rm L$ versus delivered voltage $U_\rm L$.
- Diagram below: Reference quantities over delivered voltage $U_\rm L$.

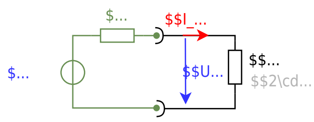
The two powers are defined as follows:

- source power: $\, \, \large{ P_\rm S = U_0 \cdot I_\rm L} $
- consumer power: $\large{ P_\rm L = U_\rm L \cdot I_\rm L} $

1. Both power $P_\rm S$ and $P_\rm L$ are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance $R_\rm L=0$. In this case, all the power flows out through the internal resistor. The efficiency drops to $0~\%$. This is the case, for example, with a battery shorted by a wire.
2. If the load resistance becomes just as large as the internal resistance $R_\rm L=R_\rm i$, the result is a voltage divider where the load voltage becomes just half the open circuit voltage: $U_\rm L = {{1}\over{2}}\cdot U_{\rm OC}$. On the other hand, the current is also half the short-circuit current $I_\rm L=I_{\rm SC}$, since the resistance at the ideal voltage source is twice that in the short-circuit case.
3. If the load resistance becomes high impedance $R_{\rm L}\rightarrow\infty$, less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches $100~\%$ for $R_{\rm L}\rightarrow\infty$.

Fig. 3: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram





The whole context can be investigated in this Simulation with a resistor or this one with a variable load.

The Efficiency

To understand the lower diagram in figure 3, the definition equations of the two reference quantities shall be described here again:

The **efficiency** \$\eta\$ describes the delivered power (consumer power) concerning the supplied power (power of the ideal source): \begin{align*} \eta = {{P_{\rm out}}\over{P_{\rm in}}} = {{R_{\rm L}\cdot I_{\rm L}^2}\over{(R_{\rm L}+R_{\rm i}) \cdot I_{\rm L}^2}} \quad \rightarrow \quad \boxed{ \eta = {{R_{\rm L}} \over {R_{\rm L}+R_{\rm i}}} } \end{align*}

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards \$\eta \rightarrow 100\%\$. This situation close to (1.) in figure 3.

Application:

1. In power engineering \$\eta \rightarrow 100\%\$ is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load \$R_{\rm L} \gg R_{\rm i} \$.

The **utilization rate** ε describes the delivered power $P_{\rm out}$ concerning the maximum possible power $P_{\rm in, max}$ of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case: \begin{align*} \varepsilon = {{P_{\rm out}}\over{P_{\rm in, max}}} = {{R_{\rm L} \cdot I_{\rm L}^2}\over{{U_0^2}\over{R_i}}} = {{R_{\rm L}\cdot R_{\rm i} \cdot I_{\rm L}^2}\over {U_0^2}} = {{R_{\rm L}\cdot R_{\rm i} \cdot \left({{U_0}\over {R_{\rm L}+R_{\rm i}}}\right)^2} \over {U_0^2}} \quad \rightarrow \quad \boxed{\varepsilon = {{R_{\rm L}\cdot R_{\rm i}} \over {(R_{\rm L}+R_{\rm i})^2}} = {{R_{\rm L}} \over {(R_{\rm L}+R_{\rm i})}}\cdot {{R_{\rm i}} \over {(R_{\rm L}+R_{\rm i})}}} \end{align*}

The Utilization Rate

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in figure 3. For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum: $\varepsilon = 25~\%$.

Application:

1. In communications engineering the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this application note for near field communication.
2. Furthermore, also for photovoltaic cells one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also here explained in a German video.

Exercise

Given: $U_0=12.0~\rm V$, $R_{\rm i}=0.50~\Omega$, $R_{\rm L}=5.0~\Omega$. **Find:** $U_{\rm L}$, $I_{\rm L}$, $P_{\rm L}$, η, ε.

\begin{align*} I_{\rm L} &= \frac{12.0~{\rm V}}{0.50~\Omega+5.0~\Omega} = 2.182~{\rm A} \\ U_{\rm L} &= I_{\rm L} R_{\rm L}= 2.182~{\rm A}\cdot 5.0~\Omega = 10.91~{\rm V} \\ P_{\rm L} &= U_{\rm L}I_{\rm L} = 10.91~{\rm V}\cdot 2.182~{\rm A}= 23.8~{\rm W} \\ P_{\rm in,max}&= \frac{U_0^2}{R_{\rm i}}=\frac{(12.0~{\rm V})^2}{0.50~\Omega}=288~{\rm W} \\ \eta &= \frac{R_{\rm L}}{R_{\rm i}+R_{\rm L}}=\frac{5.0~\Omega}{5.50~\Omega}=0.909=90.9~\% \\ \varepsilon &= \frac{R_{\rm L}R_{\rm i}}{(R_{\rm L}+R_{\rm i})^2}=\frac{5.0\cdot 0.50}{(5.50)^2}=0.0826=8.26~\% \end{align*}

Interpretation: very **efficient** (small $R_{\rm i}$) but using only **8.26 %** of the source's ideal maximum capability $U_0^2/R_{\rm i}$—which is fine for power engineering aims.

Power-flow chains (series stages)

The usable (= outgoing) $P_{\rm O}$ power of a real system is always smaller than the supplied (incoming) power $P_{\rm I}$. This is due to the fact, that there are additional losses in reality. The difference is called power loss $P_{\rm loss}$. It is thus valid:

$P_{\rm I} = P_{\rm O} + P_{\rm loss}$

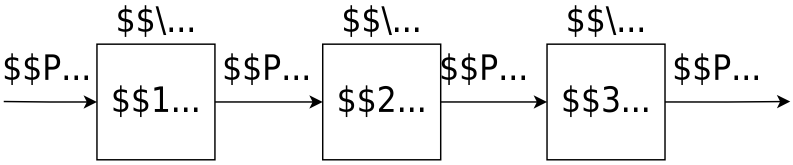
Instead of the power loss $P_{\rm loss}$, the efficiency η is often given:

$\boxed{\eta = {{P_{\rm O}}\over{P_{\rm I}}}\overset{!}{<} 1}$

For cascaded conversions (cf. figure 4), the **overall efficiency is the product** of stage efficiencies:

$\boxed{\eta = {{P_{\rm O}}\over{P_{\rm I}}} = {\not{P_{1}}\over{P_{\rm I}}}\cdot {\not{P_{2}}\over \not{P_{1}}}\cdot {{P_{\rm O}}\over \not{P_{2}}} = \eta_1 \cdot \eta_2 \cdot \eta_3}$

Fig. 4: Power flow diagram



Exercises

Exercise 7.1 Efficiency vs. maximum power (match or not?)

A source has $U_0=9.0~\rm V$, $R_{\rm i}=1.0~\Omega$.

1. (a) Choose $R_{\rm L}=9.0~\Omega$. Compute $I_{\rm L}$, $U_{\rm L}$, $P_{\rm L}$, η, ε.
2. (b) Choose $R_{\rm L}=1.0~\Omega$. Repeat. Which choice maximizes $P_{\rm L}$? Which yields higher η?

Strategy: use the boxed formulas in this block; for (b) note $R_{\rm L}=R_{\rm i} \Rightarrow \eta=50~\%$.

Exercise 7.2 Power-flow chain (product of efficiencies)

A battery (stage 1) feeds a DC/DC converter (stage 2) which feeds a sensor (stage 3). Their efficiencies are $\eta_1=0.93$, $\eta_2=0.90$, $\eta_3=0.80$.

1. Compute $\eta_{\rm total}$.
2. If the battery provides $5.0~\rm W$, what power reaches the sensor?

Exercise 7.3 — Simplify by Thevenin/Norton (preparation for Block 08)

Simplify the following circuits (*NT* for Norton, *TT* for Thevenin) to a single source plus $R_{\rm i}$, then compute $U_{\rm L}$ and η for a given $R_{\rm L}$. Fig. ##: Simplification by Norton / Thevenin theorem

a) NT

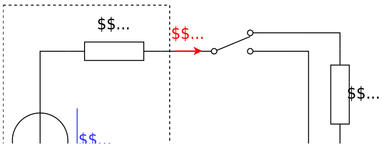
Tip: Short ideal voltage sources and open ideal current sources to determine the internal resistance.

Exercise 3.3.2 Internal resistances and Efficieny

For the company „HHN Mechatronics & Robotics" you shall analyze a competitor product: a simple drilling machine. This contains a battery pack, some electronics, and a motor. For this consideration, the battery pack can be treated as a linear voltage source with $U_{\rm s} = ~11 V$ and internal resistance of $R_{\rm i} = 0.1 ~\Omega$. The used motor shall be considered as an ohmic resistance $R_{\rm m} = 1 ~\Omega$.

The drill has two speed-modes:

1. max power: here, the motor is directly connected to the battery.
2. reduced power: in this case, a shunt resistor $R_{\rm s} = 1 ~\Omega$ is connected in series to the motor.



Tasks:

1. Calculate the input and output power for both modes.
2. What are the efficiencies for both modes?
3. Which value should the shunt resistor R_s have, when the reduced power should be exactly half of the maximum power?
4. Your company uses the reduced power mode instead of the shunt resistor $R_{\rm s}$ multiple diodes in series D, which generates a constant voltage drop of $U_D = 2.8 ~V$.
   What are the input and output power, such as the efficiency in this case?

You can check your results for the currents, voltages, and powers with the following simulation:

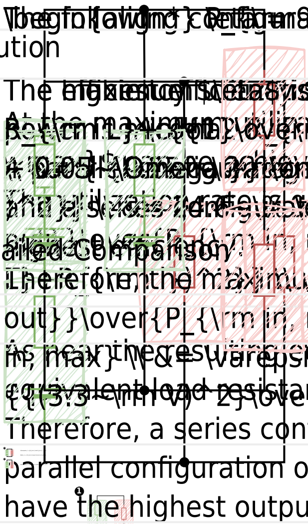
Exercise E3.3.3 Power of two pole components

Two heat resistances (both with $R_{\rm L} = 0.5 ~\Omega$) shall be supplied with two lithium-ion-batteries (both with $U_{\rm S} = 3.3 ~\rm V$, $R_{\rm i} = 0.1 ~\Omega$).

Result

Solution

1. What are the possible ways to connect these components?



Solution

The following configuration has the highest output power.

Detailed Comparison

Therefore, the maximum power is: \begin{align*} \varepsilon &= {{P_{\rm out}}\over{P_{\rm in, max}}} \\ \rightarrow P_{\rm out} &= \varepsilon \cdot P_{\rm in, max} \\ &= \varepsilon \cdot {{U_S^2}\over{R_{\rm i}}} \\ &= 24.6~\% \cdot {{(3.3~\rm V)^2}\over{0.1~\Omega}} \\ \end{align*}

As near the resulting equivalent internal resistance approaches the resulting equivalent load resistance, as higher the utilization rate ε will be.

Therefore, a series configuration of the batteries ($2 R_{\rm i} = 0.2~\Omega$) and a parallel configuration of the load (${{1}\over{2}} R_{\rm L}= 0.25~\Omega$) will have the highest output.

Summary

1. Real sources: $U_{\rm L}=U_0 \dfrac{R_{\rm L}}{R_{\rm i}+R_{\rm L}}$, $I_{\rm L}=\dfrac{U_0}{R_{\rm i}+R_{\rm L}}$; $P_{\rm L}=\dfrac{U_0^2 R_{\rm L}}{(R_{\rm i}+R_{\rm L})^2}$.
2. **Efficiency**: $\displaystyle \eta=\dfrac{R_{\rm L}}{R_{\rm i}+R_{\rm L}}$; maximize by $R_{\rm L}\gg R_{\rm i}$ (power engineering).
3. **Utilization rate**: $\displaystyle \varepsilon=\dfrac{R_{\rm L}R_{\rm i}}{(R_{\rm L}+R_{\rm i})^2}$; peak $\varepsilon_{\max}=25~\%$ at $R_{\rm L}=R_{\rm i}$ (maximum power transfer; $\eta=50~\%$).
4. **Chain efficiencies** multiply: $\eta_{\rm total}=\prod \eta_i$.
5. Thevenin/Norton help to **separate** source figures (U_0, $R_{\rm i}$) from the load and to reuse the same formulas.
6. **Max efficiency η**: $R_{\rm L}\rightarrow\infty$ (relative to $R_{\rm i}$) → small current, small loss.
7. **Max delivered power $P_{\rm L}$**: $R_{\rm L}=R_{\rm i}$ (impedance matching). See also impedance matching and Maximum Power Point Tracking (MPPT) for PV systems.

Common pitfalls checklist

1. Forgetting **units** in intermediate results (always write $x = \text{number} \times \text{unit}$).
2. Mixing up **goals**: high η vs. high $P_{\rm L}$ lead to **different** $R_{\rm L}$.
3. Using **ideal source** formulas for a **real** source (always include $R_{\rm i}$).
4. Ignoring the **sign convention** when interpreting $P=U\cdot I$ (source vs. load).