

# Block 07 — Power-relevant Figures

## Student Group

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# Block 07 — Power-relevant figures

## Learning objectives

- Define and compute **input/output power, losses, efficiency**  $\eta$  and **utilization rate**  $\varepsilon$  for DC sources and loads.
- Use the **real source model** with internal resistance  $R_{\text{i}}$  to compute operating point  $(U_{\text{L}}, I_{\text{L}})$ ,  $P_{\text{L}}$  and  $P_{\text{loss}}$ .
- Understand the different design goals:
  1. **High efficiency** (power engineering):  $R_{\text{L}} \gg R_{\text{i}}$ .
  2. **Maximum power transfer** (communications):  $R_{\text{L}} = R_{\text{i}}$ .
- Combine efficiencies along a **power-flow chain**.
- Relate these figures to **Thevenin/Norton** equivalents and the **loaded voltage divider**.

## Preparation at Home

And again:

- Please read through the following chapter.
- Also here, there are some clips for more clarification under 'Embedded resources'.

For checking your understanding please do the following exercises:

- 7.1
- E3.3.3

## 90-minute plan

1. Warm-up (8 min): recall passive/active sign convention; quick unit check for  $P=U \cdot I$ .
2. Core concepts (35 min): real source model; definitions of  $\eta$  and  $\varepsilon$ ; design goals; chain efficiency.
3. Worked example (10 min): battery + internal resistance + load.
4. Two-port view & loaded divider (12 min): quick Thevenin/Norton recap; loaded divider formulas.
5. Practice (20 min): 3 short exercises (see panels below).
6. Wrap-up (5 min): summary + pitfalls.

## Conceptual overview

1. Real sources are modeled by an **ideal source** plus **internal resistance**  $R_{\text{i}}$ ; the terminal voltage **drops under load**.
2. **Efficiency**  $\eta$  compares **delivered** to **drawn** power. In the simple DC source-load case, 
$$\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}$$
 (dimensionless).

High-efficiency design wants  $R_L \gg R_i$ .

3. **Utilization rate**  $\epsilon$  compares delivered power to the **maximum** available from the ideal source: 
$$\epsilon = \frac{R_L R_i}{(R_L + R_i)^2}$$
. It peaks at  $R_L = R_i$  with  $\epsilon_{\max} = 25\%$ . This is the **maximum power transfer** condition.
4. Different goals  $\rightarrow$  different  $R_L$ :
  - **Power engineering**: maximize  $\epsilon \rightarrow R_L \gg R_i$ .
  - **Communications** (matching, antennas, RF): maximize  $P_L \rightarrow R_L = R_i$ ,  $\epsilon = 50\%$ .

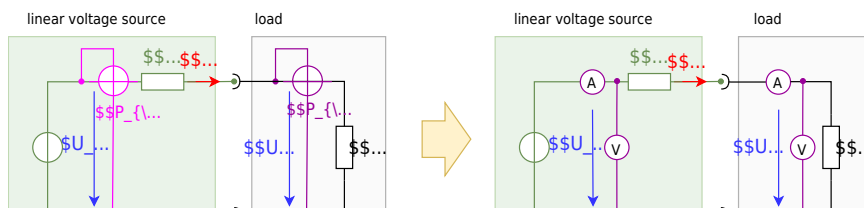
## Core content

### Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In [figure 1](#) the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source  $P_S$  and the input power of the load  $P_L$ .

Fig. 1: Power measurement on linear voltage source



## Power and Characteristics in Diagrams

The simulation in [figure 2](#) shows the following:

- The circuit with linear voltage source ( $U_0$  and  $R_{\text{int}}$ ), and a resistive load  $R_{\text{L}}$ .
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor  $R_{\text{S}}$  (English: shunt) and a voltage measurement for  $U_{\text{S}}$ . The power is then:  $P_{\text{L}} = \frac{1}{R_{\text{S}}} \cdot U_{\text{S}} \cdot U_{\text{L}}$ .
- in the oscilloscope section (below).
  - On the left is the power  $P_{\text{L}}$  plotted against time in a graph.
  - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance  $R_{\text{L}}$ , with which the value of the load resistance  $R_{\text{L}}$  can be changed.

Now try to vary the value of the load resistance  $R_{\text{L}}$  (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 2: power adjustment

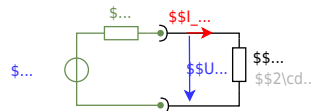
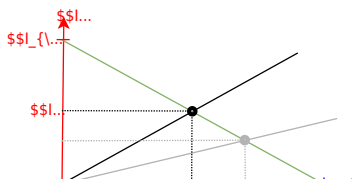
[figure 3](#) shows three diagrams:

- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power  $P_{\text{S}}$  and consumer power  $P_{\text{L}}$  versus delivered voltage  $U_{\text{L}}$ .
- Diagram below: Reference quantities over delivered voltage  $U_{\text{L}}$ .

The two powers are defined as follows:

- source power:  $P_{\text{S}} = U_0 \cdot I_{\text{L}}$
  - consumer power:  $P_{\text{L}} = U_{\text{L}} \cdot I_{\text{L}}$
1. Both power  $P_{\text{S}}$  and  $P_{\text{L}}$  are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance  $R_{\text{L}}=0$ . In this case, all the power flows out through the internal resistor. The efficiency drops to 0%. This is the case, for example, with a battery shorted by a wire.
  2. If the load resistance becomes just as large as the internal resistance  $R_{\text{L}}=R_{\text{i}}$ , the result is a voltage divider where the load voltage becomes just half the open circuit voltage:  $U_{\text{L}} = \frac{1}{2} \cdot U_{\text{OC}}$ . On the other hand, the current is also half the short-circuit current  $I_{\text{L}}=I_{\text{SC}}$ , since the resistance at the ideal voltage source is twice that in the short-circuit case.
  3. If the load resistance becomes high impedance  $R_{\text{L}} \rightarrow \infty$ , less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches 100% for  $R_{\text{L}} \rightarrow \infty$ .

Fig. 3: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram



The whole context can be investigated in this [Simulation with a resistor](#) or [this one with a variable load](#).

### The Efficiency

To understand the lower diagram in [figure 3](#), the definition equations of the two reference quantities shall be described here again:

The **efficiency**  $\eta$  describes the delivered power (consumer power) concerning the supplied power (power of the ideal source): 
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{(R_{\text{L}} + R_{\text{i}}) \cdot I_{\text{L}}^2} \quad \rightarrow \quad \boxed{\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}}$$

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards  $\eta \rightarrow 100\%$ . This situation close to (1.) in [figure 3](#).

Application:

1. In power engineering  $\eta \rightarrow 100\%$  is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load  $R_{\text{L}} \gg R_{\text{i}}$ .

## The Utilization Rate

The **utilization rate**  $\varepsilon$  describes the delivered power  $P_{\text{out}}$  concerning the maximum possible power  $P_{\text{in, max}}$  of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case: 
$$\begin{aligned} \varepsilon &= \frac{P_{\text{out}}}{P_{\text{in, max}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{U_0^2 \cdot \frac{1}{R_{\text{i}}}} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot I_{\text{L}}^2}{U_0^2} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot \left(\frac{U_0}{R_{\text{L}} + R_{\text{i}}}\right)^2}{U_0^2} \quad \rightarrow \quad \boxed{\varepsilon = \frac{R_{\text{L}} \cdot R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2}} \end{aligned}$$

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in [figure 3](#). For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum:  $\varepsilon = 25\%$ .

Application:

1. In [communications engineering](#) the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this [application note for near field communication](#).
2. Furthermore, also for [photovoltaic cells](#) one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also [here](#) explained in a German video.

### Exercise

**Given:**  $U_0 = 12.0 \text{ V}$ ,  $R_{\text{i}} = 0.50 \text{ }\Omega$ ,  $R_{\text{L}} = 5.0 \text{ }\Omega$ . **Find:**  $U_{\text{L}}$ ,  $I_{\text{L}}$ ,  $P_{\text{L}}$ ,  $\eta$ ,  $\varepsilon$ .

$$\begin{aligned} I_{\text{L}} &= \frac{12.0 \text{ V}}{0.50 \text{ }\Omega + 5.0 \text{ }\Omega} = 2.182 \text{ A} \\ U_{\text{L}} &= I_{\text{L}} \cdot R_{\text{L}} = 2.182 \text{ A} \cdot 5.0 \text{ }\Omega = 10.91 \text{ V} \\ P_{\text{L}} &= U_{\text{L}} \cdot I_{\text{L}} = 10.91 \text{ V} \cdot 2.182 \text{ A} = 23.8 \text{ W} \\ P_{\text{in, max}} &= \frac{U_0^2}{R_{\text{i}}} = \frac{(12.0 \text{ V})^2}{0.50 \text{ }\Omega} = 288 \text{ W} \\ \eta &= \frac{R_{\text{L}}}{R_{\text{i}} + R_{\text{L}}} = \frac{5.0 \text{ }\Omega}{5.50 \text{ }\Omega} = 0.909 = 90.9\% \\ \varepsilon &= \frac{R_{\text{L}} \cdot R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2} = \frac{5.0 \cdot 0.50}{(5.50)^2} = 0.0826 = 8.26\% \end{aligned}$$

Interpretation: very **efficient** (small  $R_{\text{i}}$ ) but using only **8.26 %** of the source's ideal maximum capability  $U_0^2/R_{\text{i}}$ —which is fine for power engineering aims.

## Power-flow chains (series stages)

The usable (= outgoing)  $P_{\text{O}}$  power of a real system is always smaller than the supplied (incoming) power  $P_{\text{I}}$ . This is due to the fact, that there are additional losses in reality. The difference is called power loss  $P_{\text{loss}}$ . It is thus valid:

$$P_{\text{I}} = P_{\text{O}} + P_{\text{loss}}$$

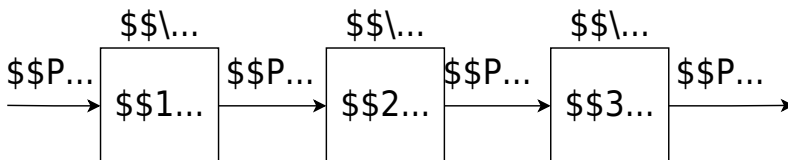
Instead of the power loss  $P_{\text{loss}}$ , the efficiency  $\eta$  is often given:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}}} \overset{!}{< 1}$$

For cascaded conversions (cf. [figure 4](#)), the **overall efficiency is the product** of stage efficiencies:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} = \frac{P_{\text{O}_1}}{P_{\text{I}_1}} \cdot \frac{P_{\text{O}_2}}{P_{\text{I}_2}} \cdot \frac{P_{\text{O}_3}}{P_{\text{I}_3}} = \eta_1 \cdot \eta_2 \cdot \eta_3}$$

Fig. 4: Power flow diagram



## Exercises

### Exercise 7.1 Efficiency vs. maximum power (match or not?)

A source has  $U_0 = 9.0 \text{ V}$ ,  $R_{\text{i}} = 1.0 \text{ }\Omega$ .

- (a) Choose  $R_{\text{L}} = 9.0 \text{ }\Omega$ . Compute  $I_{\text{L}}$ ,  $U_{\text{L}}$ ,  $P_{\text{L}}$ ,  $\eta$ ,  $\varepsilon$ .
- (b) Choose  $R_{\text{L}} = 1.0 \text{ }\Omega$ . Repeat.

Which choice maximizes  $P_{\text{L}}$ ? Which yields higher  $\eta$ ?

**Strategy:** use the boxed formulas in this block; for (b) note  $R_{\text{L}} = R_{\text{i}}$   
 $\rightarrow \eta = 50\%$ .

### Exercise 7.2 Power-flow chain (product of efficiencies)

A battery (stage 1) feeds a DC/DC converter (stage 2) which feeds a sensor (stage 3). Their efficiencies are  $\eta_1 = 0.93$ ,  $\eta_2 = 0.90$ ,  $\eta_3 = 0.80$ .

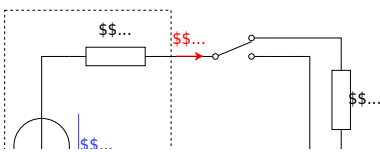
- Compute  $\eta_{\text{total}}$ .
- If the battery provides  $5.0 \text{ W}$ , what power reaches the sensor?

### Exercise 3.3.2 Internal resistances and Efficiency

For the company „HHN Mechatronics & Robotics“ you shall analyze a competitor product: a simple drilling machine. This contains a battery pack, some electronics, and a motor. For this consideration, the battery pack can be treated as a linear voltage source with  $U_{\text{s}} = 11 \text{ V}$  and internal resistance of  $R_{\text{i}} = 0.1 \text{ }\Omega$ . The used motor shall be considered as an ohmic resistance  $R_{\text{m}} = 1 \text{ }\Omega$ .

The drill has two speed-modes:

- max power: here, the motor is directly connected to the battery.
- reduced power: in this case, a shunt resistor  $R_{\text{s}} = 1 \text{ }\Omega$  is connected in series to the motor.



Tasks:

1. Calculate the input and output power for both modes.
2. What are the efficiencies for both modes?
3. Which value should the shunt resistor  $R_S$  have, when the reduced power should be exactly half of the maximum power?
4. Your company uses the reduced power mode instead of the shunt resistor  $R_S$  multiple diodes in series  $D$ , which generates a constant voltage drop of  $U_D = 2.8 \sim V$ .  
What are the input and output power, such as the efficiency in this case?

You can check your results for the currents, voltages, and powers with the following simulation:

Exercise E3.3.3 Power of two pole components

What is the ratio of the maximum efficiency  $\eta_{max}$  of a power supply  $P_{max}$  at the loads two lithium-batteries (both with  $U_S = 3.3 \sim V$ ,  $R_i = 0.1 \sim \Omega$ ).

Solution

The following circuit diagram shows the possible ways to connect these components.

The highest efficiency is achieved when the output power is maximized. At the maximum utilization rate  $\eta_{max} = P_{out} / P_{in} = 0.25$  the maximum power  $P_{max}$  is achieved. The utilization rate is given by  $\eta = P_{out} / P_{in} = 0.25$  will have the

Details of the maximum power transfer theorem:  $P_{out} = \epsilon \cdot P_{in, max}$  where  $\epsilon = \frac{R_L}{R_L + R_i}$ . For the maximum power transfer, the load resistance  $R_L$  must be equal to the internal resistance  $R_i$ . In this case, the resulting equivalent load resistance, as higher the utilization rate  $\epsilon$  will be. Therefore, a series configuration of the batteries ( $R_i = 0.2 \Omega$ ) and a parallel configuration of the load ( $\frac{1}{2} R_L = 0.25 \Omega$ ) will have the highest output.



### Exercise E1 Efficiency

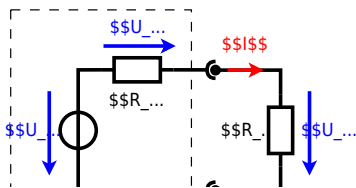
(written test, approx. 14 % of a 60-minute written test, SS2023)

2. (34.4%) A battery with an internal resistance  $R_i = 0.2 \Omega$  and a nominal voltage  $U_N = 3.5 V$ . The battery shall provide energy for a device with an load resistance of  $R_L = 2 \Omega$ . The following values are given from the battery data sheet:

Capacity:  $Q = 2.6 Ah$   
 Discharge current:  $I_{Dis, max} = 3.5 A$   
 Efficiency:  $\eta = 0.95$

.. Derive an equivalent circuit diagram with the internal resistance and an external load. Label voltages and currents.

Result:  $\eta = \frac{P_{out}}{P_{in}} = \frac{U_L \cdot I_L}{U_N \cdot I_N} = \frac{U_L \cdot I_L}{U_N \cdot (I_L + I_{R_i})} = \frac{U_L \cdot I_L}{U_N \cdot I_L (1 + \frac{R_i}{R_L})} = \frac{U_L}{U_N (1 + \frac{R_i}{R_L})} = \frac{2.6 V}{3.5 V (1 + \frac{0.2 \Omega}{2 \Omega})} = 0.95$



## Summary

1. Real sources:  $U_L = U_0 \frac{R_L}{R_i + R_L}$ ,  $I_L = \frac{U_0}{R_i + R_L}$ ;  $P_L = \frac{U_0^2 R_L}{(R_i + R_L)^2}$ .
2. **Efficiency:**  $\eta = \frac{R_L}{R_i + R_L}$ ; maximize by  $R_L \gg R_i$  (power engineering).
3. **Utilization rate:**  $\epsilon = \frac{R_L R_i}{(R_L + R_i)^2}$ ; peak  $\epsilon_{\max} = 25\%$  at  $R_L = R_i$  (maximum power transfer;  $\eta = 50\%$ ).
4. **Chain efficiencies** multiply:  $\eta_{\text{total}} = \prod \eta_i$ .
5. Thevenin/Norton help to **separate** source figures ( $U_0$ ,  $R_i$ ) from the load and to reuse the same formulas.
6. **Max efficiency  $\eta$ :**  $R_L \rightarrow \infty$  (relative to  $R_i$ )  $\rightarrow$  small current, small loss.
7. **Max delivered power  $P_L$ :**  $R_L = R_i$  (impedance matching).  
See also [impedance matching](#) and [Maximum Power Point Tracking \(MPPT\)](#) for PV

systems.

## Common pitfalls checklist

1. Forgetting **units** in intermediate results (always write  $x = \text{number} \times \text{unit}$ ).
2. Mixing up **goals**: high  $\eta$  vs. high  $P_{\text{L}}$  lead to **different**  $R_{\text{L}}$ .
3. Using **ideal source** formulas for a **real** source (always include  $R_{\text{i}}$ ).
4. Ignoring the **sign convention** when interpreting  $P=U \cdot I$  (source vs. load).

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