Block 10 - Field Patterns of key Geometries

Student Group

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Block 10 - Field Patterns of key Geometries

Learning objectives

> By the end of this section, you will be able to:
>
> - Explain and sketch **electric field lines** for single and multiple charges; state that line **direction** follows the force on a positive test charge and line **density** indicates $|\vec{E}|$.
> - Distinguish **homogeneous** fields (e.g. ideal parallel plates) from **inhomogeneous** fields (e.g. point charge, edges) and relate $E=\frac{U}{d}$ in plate geometries.
> - State conductor boundary facts in electrostatics: $\vec{E}$ is **perpendicular** to conducting surfaces and the **interior is field-free**; surfaces are **equipotentials**.
> - Use the **superposition principle** to construct field patterns.
> - Compute $|\vec{E}|$ for a **point charge** with $\varepsilon=\varepsilon_0\varepsilon_r$: $\displaystyle |\vec{E}|=\frac{1}{4\pi\varepsilon}\frac{|Q|}{r^2}$.

Preparation at Home

And again:

- Please read through the following chapter.
- Also here, there are some clips for more clarification under 'Embedded resources'.

For checking your understanding please do the following exercise:

- 1.1.2
- 1.2.5

90-minute plan

- **Warm-up (8–10 min)**

Quick sketches: single charge, dipole, parallel plates. Poll for rules of field lines and equipotentials.

- **Concept build & demonstrations (35–40 min)**
  1. Rules for **field lines**: start at $+$, end at $-$, no intersections; density $\propto |\vec{E}|$; not particle trajectories.
  2. **Homogeneous vs. inhomogeneous**: parallel-plate region ($E=\frac{U}{d}$) vs. point/edge fields ($|\vec{E}|\sim 1/r^2$ near a point charge).
  3. **Conductors in electrostatics**: interior $E=0$, surface is an **equipotential**, $\vec{E}\perp$ surface; charge crowds near sharp curvature.
  4. **Superposition**: build dipole and two-like-charge patterns from single-charge fields.
- **Guided simulations (20–25 min)**

Move charges, toggle equipotentials, and compare line density to indicated $|\vec{E}|$; vary plate spacing d and discuss $E=\frac{U}{d}$ (units: $\rm V/m$).

- **Practice (10–15 min)**

Mini-worksheet: sketch fields for two like charges and a dipole; mark where $|\vec{E}|$ is largest; short calc: $|\vec{E}|$ at r from a charge.

- **Wrap-up (5 min)**

Summary map linking **field lines ↔ equipotentials ↔ potential difference** as bridge to capacitors and energy (next blocks).

Conceptual overview

1. **Field lines** visualize $\vec{E}$: start at $+$, end at $-$, never intersect; higher line density ⇔ larger $|\vec{E}|$; lines are **not** particle trajectories.
2. **Homogeneous fields** (ideal between large parallel plates): parallel, equally spaced lines; **inhomogeneous fields** elsewhere (e.g., point charges, edges).
3. **Conductors (electrostatics)**: $\vec{E}$ is perpendicular to the surface; interior is field-free; surface charge arranges to enforce these conditions.

- **What field lines mean:** visual aid for $\vec{E}$.
  they start on positive charge and end on negative charge; their **density** reflects the **magnitude** $|\vec{E}|$; arrows show the **force direction on a positive test charge**. Lines never intersect.
- **Homogeneous vs. inhomogeneous:** between large, parallel plates the field is approximately uniform with $E=\frac{U}{d}$;
  around localized or curved conductors and point charges the field varies with position (e.g. $|\vec{E}|\propto 1/r^2$ for a point charge).
- **Conductors (electrostatics):** inside an ideal conductor $E=0$; surfaces are equipotentials;
  $\vec{E}$ meets the surface **perpendicularly**; surface charge re-arranges to enforce these conditions and concentrates at sharp edges.
- **Superposition:** total field is the vector sum of contributions from all charges; use it to construct patterns for dipoles and multi-charge systems.

Core content

Geometric Distribution of Charges

In previous chapters, only single charges (e.g., Q_1, Q_2) were considered.

- The charge Q was previously reduced to a **point charge**.
  This can be used, for example, for the elementary charge or for extended charged objects from a large distance. The distance is sufficiently large if the ratio between the largest object extent and the distance to the measurement point P is small.

- If the charges are lined up along a line, this is referred to as a **line charge**.
  Examples of this are a straight trace on a circuit board or a piece of wire. Furthermore, this also applies to an extended charged object, which has exactly an extension that is no longer small in relation to the distance. For this purpose, the charge Q is considered to be distributed over the line. Thus, a (line) charge density ρ_l can be determined:

  $$\rho_l = {{Q}\over{l}}$$

  or, in the case of different charge densities on subsections:

  $$\rho_l = {{\Delta Q}\over{\Delta l}} \rightarrow \rho_l(l)={{\rm d}\over{{\rm d}l}} Q(l)$$

- It is spoken of as an **area charge** when the charge is distributed over an area.
  Examples of this are the floor or the plate of a capacitor. Again, an extended charged object can be considered when two dimensions are no longer small in relation to the distance (e.g. surface of the earth). Again, a (surface) charge density ρ_A can be determined:

  $$\rho_A = {{Q}\over{A}}$$

  or if there are different charge densities on partial surfaces:

  $$\rho_A = {{\Delta Q}\over{\Delta A}} \rightarrow \rho_A(A) ={{\rm d}\over{{\rm d}A}} Q(A)={{\rm d}\over{{\rm d}x}}{{\rm d}\over{{\rm d}y}} Q(A)$$

- Finally, a **space charge** is the term for charges that span a volume.
  Here, examples are plasmas or charges in extended objects (e.g., the doped volumes in a semiconductor). As with the other charge distributions, a (space) charge density ρ_V can be calculated here:

  $$\rho_V = {{Q}\over{V}}$$

  or for different charge density in partial volumes:

  $$\rho_V = {{\Delta Q}\over{\Delta V}} \rightarrow \rho_V(V) ={{\rm d}\over{{\rm d}V}} Q(V)={{\rm d}\over{{\rm d}x}}{{\rm d}\over{{\rm d}y}}{{\rm d}\over{{\rm d}z}} Q(V)$$

Electric Field Lines

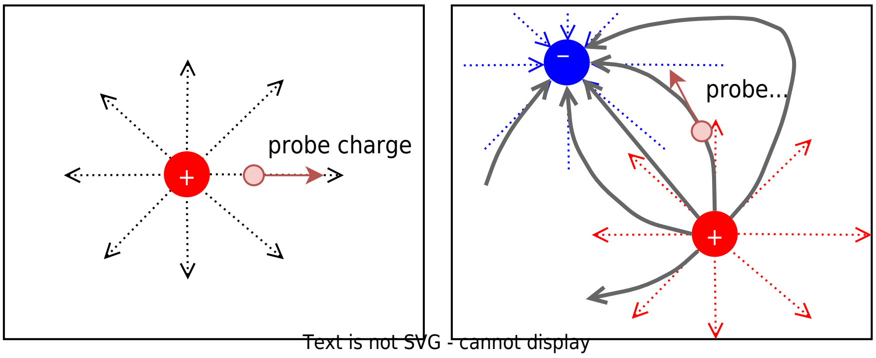
Electric field lines result from the (fictitious) path of a sample charge. Thus, also electric field lines of several charges can be determined. However, these also result from a superposition of the individual effects - i.e., electric field - at a measuring point P.

The superposition is sketched in figure 4: Two charges Q_1 and Q_2 act on the test charge q with the forces F_1 and F_2. Depending on the positions and charges, the forces vary, and so does the resulting force. The simulation also shows a single field line.

Fig. 1: examples of field lines

For a full picture of the field lines between charges, one has to start with a single charge. The in- and outgoing lines on this charge are drawn equidistant from the charge. This is also true for the situation with multiple charges. However, there, the lines are not necessarily run radially anymore. The test charge is influenced by all the single charges, and therefore, the field lines can get bent.

Fig. 2: examples of field lines



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In figure 3 the field lines are shown. The additional “equipotential lines” will be discussed later and can be deactivated by clearing the checkmark `Show Equipotentials`. Try the following in the simulation:

- Get accustomed to the simulation. You can…
  - … move the charges by drag and drop.
  - … add another Charge with `Add » Add Point Charge`.
  - … delete components with a right click on them and `delete`
- Where is the density of the field lines higher?
- How does the field between two positive charges look? How does it look between two different charges?

Fig. 3: examples of field lines

Fig. 4: examples of field lines in 3D

Note:

1. The electrostatic field is a source field. This means there are sources and sinks.
2. From the field line diagrams, the following can be obtained:
   1. Direction of the field ($\hat{=}$ parallel to the field line).
   2. Magnitude of the field ($\hat{=}$ number of field lines per unit area).
3. The magnitude of the field along a field line is usually not constant.

Note:

Field lines have the following properties:

- The electric field lines have a beginning (at a positive charge) and an end (at a negative charge).
- The direction of the field lines represents the direction of a **force onto a positive test charge**.
- There are **no closed field lines** in electrostatic fields. The reason for this can be explained by considering the energy of the moved particle (see later subchapters).
- Electric **field lines cannot cut** each other: This is based on the fact that the direction of the force at a cutting point would not be unique.
- The field lines are **always perpendicular to conducting surfaces**
- The **inside of a conducting component is always field-free**.
- The density of the field lines is a measure for the electric field density.

Types of Fields depending on the Charge Distribution

There are two different types of fields:

In **homogeneous fields**, magnitude and direction are constant throughout the field range. This field form is idealized to exist within plate capacitors. e.g., in the plate capacitor (figure 5), or the vicinity of widely extended bodies.

Here, the electric field E is given as: $E = {{U}\over{d}}$

Fig. 5: Field lines of a homogeneous field

For **inhomogeneous fields**, the magnitude and/or direction of the electric field changes from place to place. This is the rule in real systems, even the field of a point charge is inhomogeneous (figure 6).

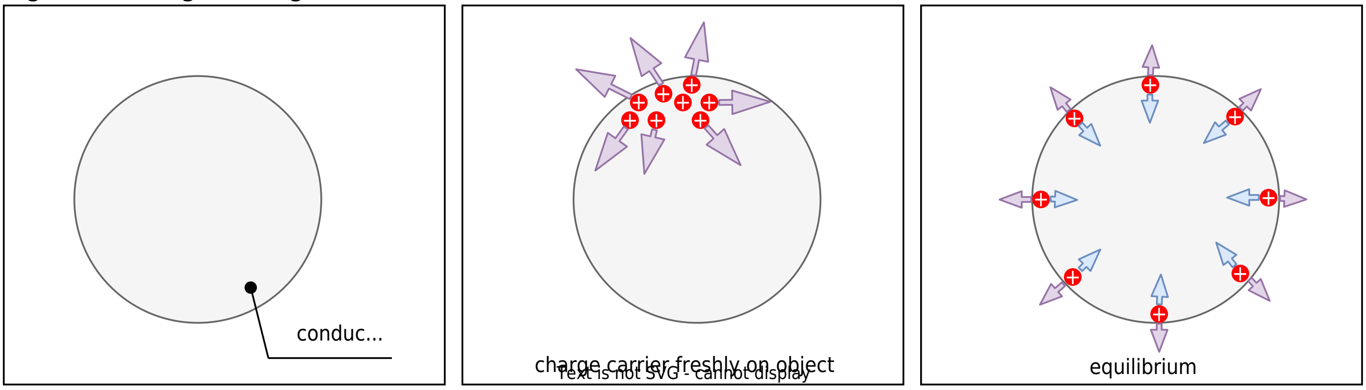
For the given example of a cylindrical configuration woth tha radius r, the electric field E is given as: $E \sim {{1}\over{r}}$

Fig. 6: Field lines of an inhomogeneous field

Stationary Situation of a charged conducting Object (without an external Field)

In the first thought experiment, a conductor (e.g., a metal plate) is charged, see figure 7. The additional charges create an electric field. Thus, a resultant force acts on each charge. The causes of this force are the electric fields of the surrounding electric charges. So the charges repel and move apart.

Fig. 7: Viewing a charged metal ball



The movement of the charge continues until a force equilibrium is reached. In this steady state, there is no longer a resultant force acting on the single charge. In figure 7 this can be seen on the right: the repulsive forces of the charges are counteracted by the attractive forces of the atomic shells.

Results:

- The charge carriers are distributed on the surface.
- Due to the dispersion of the charges, the interior of the conductor is free of fields.
- All field lines are perpendicular to the surface. Because: if they were not, there would be a parallel component of the field, i.e., along the surface. Thus, a force would act on charge carriers, and they would move accordingly.

Educational Task - Why is there a discharge at pointy ends of conductors?

Point discharge is a well-known phenomenon, which can be seen as corona discharge on power lines (where it also creates the summing sound) or is used in spark plugs. The phenomenon addresses the effect that there are many more charges at the corners and edges of a conductor. But why is that so? For this, it is feasible to try to calculate the charge density at different spots of a conductor.

Fig. 8: field of a pointy object (field line density is not correct)

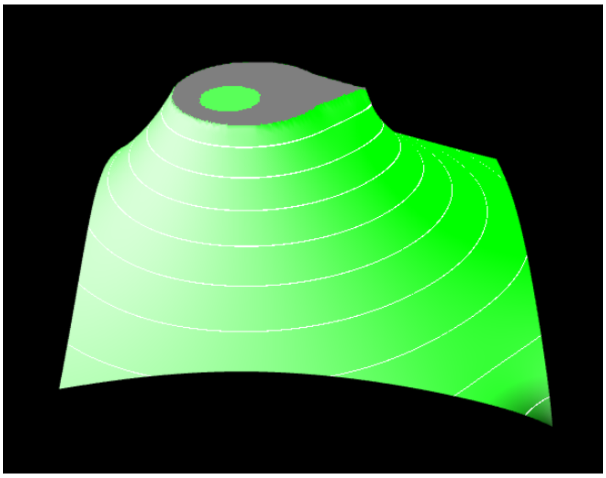
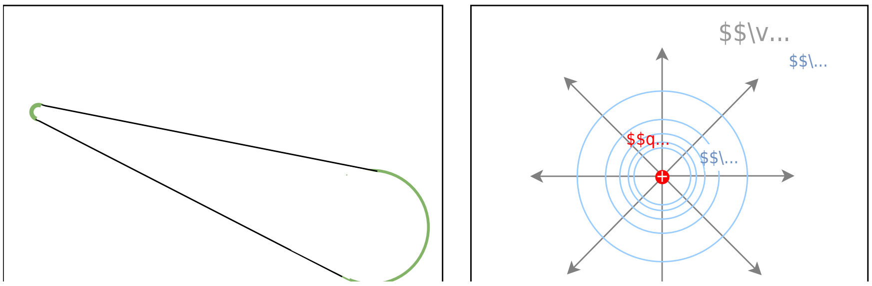


Fig. 9: examples for an arbitrarily formed conductor



In the figure 9 an example of a charged and "pointy" conductor is given in image (a). The surface of the conductor is always at the same potential. To cope with this complex shape and the desired charge density, the following path shall be taken:

1. It is good to first calculate the potential field of a point charge.
   For this calculate $U_{ \rm CG} = \int_{ \rm C}^{ \rm G} \vec{E} \cdot {\rm d} \vec{s}$ with $\vec{E} ={{1} \over {4\pi\cdot\varepsilon}} \cdot {{q} \over {r^2}} \cdot \vec{e}_r $, where $\vec{e}_r$ is the unit vector pointing radially away, ${ \rm C}$ is a point at distance r_0 from the charge and ${ \rm G}$ is the ground potential at infinity.
2. Compare the field and the potentials of the different spherical conductors in figure 9, image (b).
   1. Are there differences for the electric field $\vec{E}$ outside the spherical conductors? Are the potentials on the surface the same?
   2. What can be conducted for the field of the three situations in (b) and (d), when the total charge on the surface is considered to be always the same?
3. For spherical conductors, the surface charge density is constant. Given that this charge density leads to the overall charge q, how does ϱ_A depend on the radius r of a sphere?
4. Now, the situation in (c) shall be considered. Here, all components are conducting, i.e., the potentials on the surface are similar. Both spheres shall be considered to be as far away from each other, so that they show an undisturbed field near their surfaces. In this case, charges on the surface of the curvature to the left and the right represent the same situation as in (a). For the next step, it is important that by this, the potentials of the left sphere with q_1 and r_1 and the right sphere with q_2 and r_2 are the same.

1. Set up this equality formula based on the formula for the potential from question 1.
2. Insert the relationship for the overall charges q_1 and q_2 based on the surface charge densities ϱ_{A1} and ϱ_{A2} of a sphere and their radii r_1 and r_2.
3. What is the relationship between the bending of the surface and the charge density?

Dielectric strength

In Block03 we had a short look on conductivity of matter.
Here, we want to have again a look onto isolators.

- The ability to insulate is dependent on the material.
- If a maximum electric field E_0 is exceeded, the insulating ability is eliminated.
  - One says: The insulator breaks down. This means that above this electric field, a current can flow through the insulator.
  - Examples are: Lightning in a thunderstorm, ignition spark, glow lamp in a phase tester
  - The maximum electric field E_0 is referred to as **dielectric strength** (in German: *Durchschlagfestigkeit* or *Durchbruchfeldstärke*).
  - E_0 depends on the material (see table 1), but also on other factors (temperature, humidity, …).

Material	Dielectric strength E_0 in ${ \rm kV/mm}$
air	$\rm 0.1...0.3$
SF6 gas	$\rm 8$
insulating oils	$\rm 5...30$
vacuum	$\rm 20...30$
quartz	$\rm 30...40$
PP, PE	$\rm 50$
PS	$\rm 100$
distilled water	$\rm 70$

Tab. 1: Dielectric strength

Common pitfalls

- Treating field lines as **charge paths**: they are drawings of direction/magnitude of $\vec{E}$, **not** particle trajectories.
- Forgetting the **reference charge sign**: line arrows indicate the force on a **positive** test charge; forces on electrons point opposite to the arrows.
- Mixing up **equipotentials** and field lines: equipotentials are everywhere **perpendicular** to field lines; they do **not** indicate current.
- Assuming the plate field is always perfectly uniform: edge effects make real plate fields **inhomogeneous** away from the central region.
- Ignoring conductor boundary conditions: in electrostatics the interior of a conductor is **field-free** and $\vec{E}$ is **normal** to the surface; any tangential $\vec{E}$ would drive charges until it vanishes.
- Confusing $\vec{E}$ with $\vec{D}$: here we use $\vec{E}$ and **permittivity**

$\varepsilon=\varepsilon_0\varepsilon_r$ for $|\vec{E}|=\frac{1}{4\pi\varepsilon}\frac{|Q|}{r^2}$.

Exercises

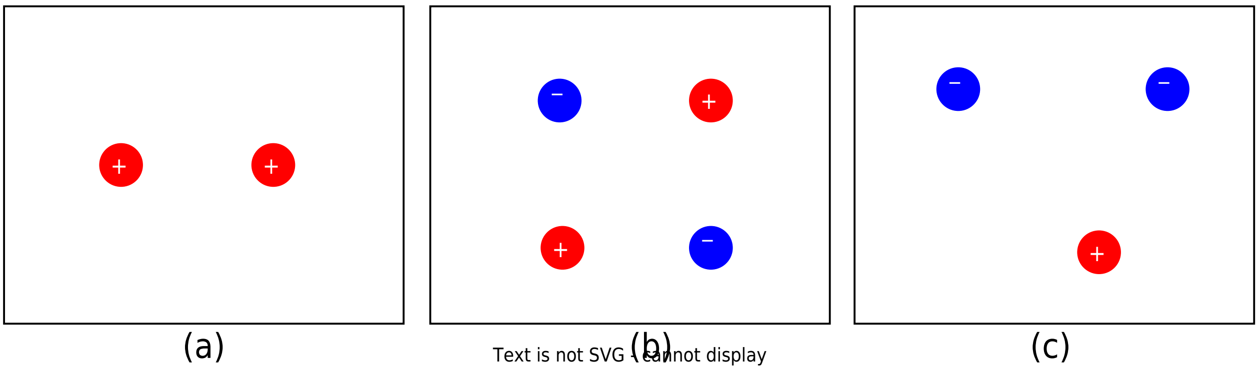
Task 1.1.2 Field lines

Sketch the field line plot for the charge configurations given in figure 10.
Note:

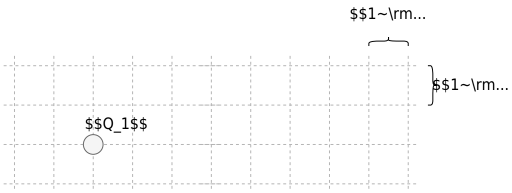
- The overlaid picture is requested.
- Make sure that it is a source field.

You can prove your result with the simulation figure 4.

Fig. 10: Task on field lines



Task 1.2.5 Forces on Charges (exam task, ca 8 % of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1=7 ~\rm{µC}$ (point charge)
$Q_2=5 ~\rm{µC}$ (point charge)
$Q_3=0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Tips for the solution

- Which equation is to be used for the force effect of charges?
- How can the distance between the two charges be determined?

Solution

\begin{align*} F_C &= {{{1} \over {4\pi\cdot\varepsilon}} \cdot {{Q_1 \cdot Q_2} \over {r^2}}} \quad && | \text{with } r=\sqrt{\Delta x^2 + \Delta y^2} \\ F_C &= {{{1} \over {4\pi\cdot\varepsilon}} \cdot {{Q_1 \cdot Q_2} \over {\Delta x^2 + \Delta y^2}}} \quad && | \text{Insert numerical values, read off distances: } \Delta x = 5~\rm{dm}, \Delta y = 3~\rm{dm} \\ F_C &= {{{1} \over {4\pi\cdot 8.854\cdot 10^{-12} ~\rm{F/m}}} \cdot {{7 \cdot 10^{-6} ~\rm{C} \cdot 5 \cdot 10^{-6} ~\rm{C}} \over { (0.5~\rm{m})^2 + (0.2~\rm{m})^2}}} \end{align*}

Result

$$\begin{align*} |\vec{F}_C| = 1.084 ~\rm{N} \rightarrow 1.1 ~\rm{N} \end{align*}$$

2. is this force attractive or repulsive?

Tips for the solution

- What force effect do equally or oppositely charged bodies exhibit on each other?

Solution

The force is repulsive because both charges have the same sign.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=100 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Tips for the solution

- Which equation is to be applied for the force action in the homogeneous field?

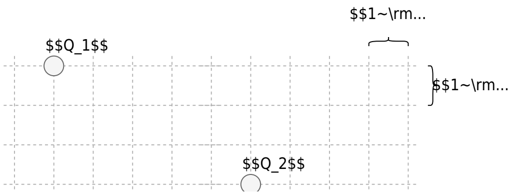
Solution

$$\begin{align*} F_C &= E \cdot Q_1 \quad && | \text{Insert numerical values} \\ F_C &= 100 \cdot 10^3 ~\rm{V/m} \cdot 7 \cdot 10^{-6} ~\rm{C} \end{align*}$$

Result

$$\begin{align*} |\vec{F}_C| = 0.7 ~\rm{N} \end{align*}$$

Task 1.2.6 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1=5 ~\rm{µC}$ (point charge)
$Q_2=-10 ~\rm{µC}$ (point charge)
$Q_3= 0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Result

\begin{align*} |\vec{F}_C| = 1.321 ~\rm{N} \rightarrow 1.3 ~\rm{N} \end{align*}

2. is this force attractive or repulsive?

Solution

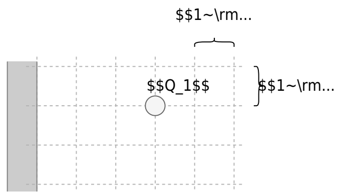
The force is repulsive because both charges have the same sign.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=500 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Result

\begin{align*} |\vec{F}_C| = 2.5 ~\rm{N} \end{align*}

Task 1.2.7 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1= 2 ~\rm{µC}$ (point charge)
$Q_2=-4 ~\rm{µC}$ (point charge)
$Q_3= 0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Result

\begin{align*} |\vec{F}_C| = 0.3595 ~\rm{N} \rightarrow 0.36 ~\rm{N} \end{align*}

2. is this force attractive or repulsive?

Solution

The force is attractive because the charges have different signs.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a

homogeneous electric field with $E_3=100 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Result

\begin{align*} |\vec{F}_C| = 0.4 ~\rm{N} \end{align*}

Exercise E2 Electrostatics I
(written test, approx. 10 % of a 120-minute written test, SS2022)

[illegible] charges E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Result

Path

- $q_0=-1 ~\rm nC$
- $q_1=-2 ~\rm nC$

Path

$E_4 = 19.97 {\rm kV}\over{\rm m}}$

  - $\vec{F}_{01} = \left({\begin{array}{cccc} 19.97 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
  - $\vec{F}_{02} = \left({\begin{array}{cccc} 28.09 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
  - $\vec{F}_{03} = \left({\begin{array}{cccc} 10.05 {~\rm \mu N} \\ 20.10 {~\rm \mu N} \\ \end{array} } \right)$

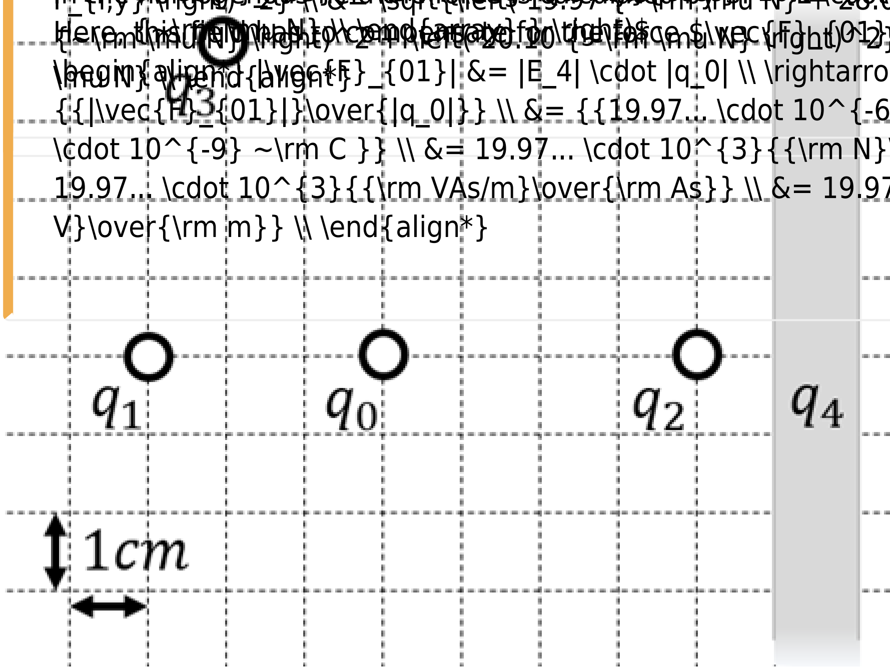
In the beginning, the area charge is $q_4=0 ~\rm nC$.
The permittivity is $\varepsilon_{\rm r}\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

With all the x and y components, we can calculate the resulting magnitude of the force with the Pythagorean theorem:

\begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(19.97 {~\rm \mu N} + 28.09 {~\rm \mu N} - 10.05 {~\rm \mu N} \right)^2 + \left(20.10 {~\rm \mu N} \right)^2} \\ &= 42.99... {~\rm \mu N} \end{align*}

In the homogenous field the force is calculated by $F = E \cdot q$.
Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:

\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{19.97... \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 19.97... \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 19.97... \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 19.97... \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, calculate the magnitude of the forces, like $\vec{F}_{01}$.
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to

$F_{01,x}$.

$$\begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 2 \cdot 10^{-9} ~\rm C}\over{(3 \cdot 10^{-2} ~\rm m)^2}} \\ &= 19.97... \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 19.97... \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 19.97... \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= 19.97... {\rm \mu N} \quad \text{(to the right)} \end{align*}$$

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$

$$\begin{align*} \vec{F}_{02} = F_{02,x} &= -28.09... {\rm \mu N} \quad \text{(to the right)} \\ \vec{F}_{03} &= -22.47... {\rm \mu N} \quad \text{(to the top left)} \\ \end{align*}$$

For $\vec{F}_{03}$, we have to calculate the x- and y-component.
This is possible, by using the angle α between the line through q_0 and q_3 and the positive x-axis (pointing to the right).
So, α has to be between $90°$ and $180°$. It can be calculated by:

$$\begin{align*} \alpha = \arctan(\rm {{-4~cm}\over{+2~cm}}) = \pi - 1.1071... = 180° - 63.4...° = 116.6...° \end{align*}$$

Based on this, the x- and y-component is:

$$\begin{align*} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... {~\rm \mu N} \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... {~\rm \mu N} \text{(to the top)} \\ \end{align*}$$

Exercise E1 Electrostatics I
(written test, approx. 8 % of a 120-minute written test, SS2024)

2. What is the magnitude of the resulting force?

3. Now the charge $q_2=0$ is set to 0. The area charge q_4 generates a homogenous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Result

Path

- $q_0=-1 ~\rm nC$
- $q_1=-5 ~\rm nC$

Path

$E_4 = 917{\rm V}\over{\rm m}$

- $\vec{F}_{01} = \left({\begin{array}{cccc} +917 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{02} = \left({\begin{array}{cccc} -1997 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{03} = \left({\begin{array}{cccc} 0 {~\rm \mu N} \\ -1123 {~\rm \mu N} \\ \end{array} } \right)$

With all the x- and y-components, we can calculate the resulting magnitude of the force with the Pythagorean Theorem:

$$\begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(+917 {~\rm \mu N} - 1997 {~\rm \mu N} \right)^2 + \left(-1123 {~\rm \mu N} \right)^2} \\ &= 1560.\;.\!.\!.\! {~\rm \mu N} \\ \end{align*}$$

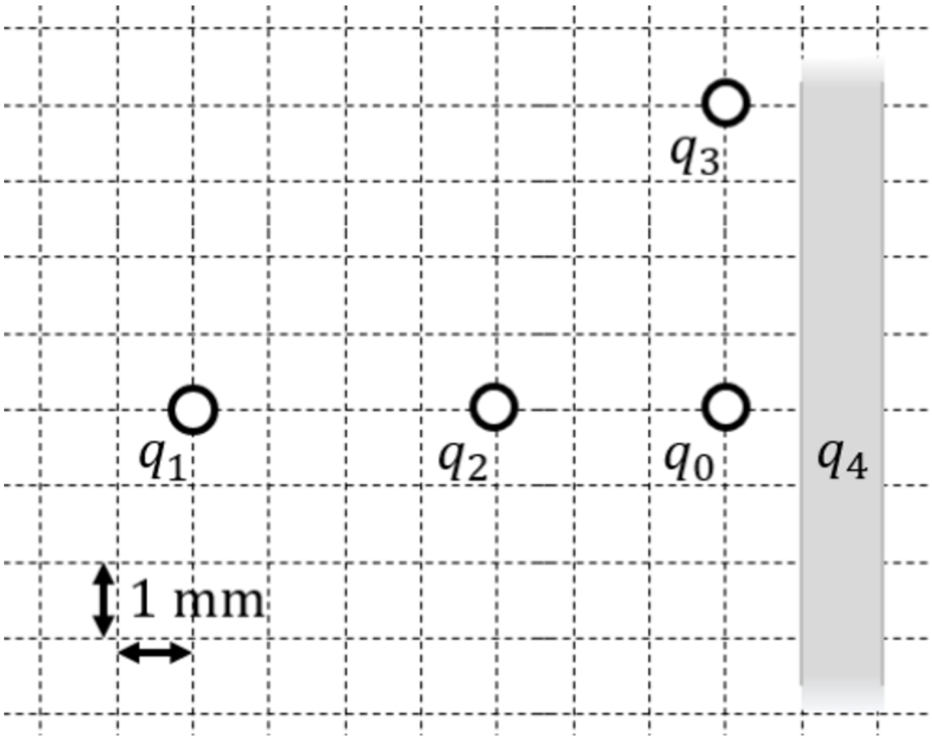
In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_{\rm r}\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

In the homogenous field the force is calculated by $F = E \cdot q$.

Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:

$$\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{917.\;.\!.\!.\! \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}$$



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, set up a coordinate system. Here, I choose x pointing to the right (positive values to the right) and y pointing upwards (positive values upwards).

Then, calculate the magnitude of the forces, like $\vec{F}_{01}$ (force on q_0 from q_1).
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 5 \cdot 10^{-9} ~\rm C}\over{(7 \cdot 10^{-3} ~\rm m)^2}} \\ &= 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= +917.\;.\!.\!.\! {\rm \mu N} \end{align*} Since both q_0 and q_1 have the same sign for their charges, they are repelling each other. Therefore, The force $\vec{F}_{01}$ points to the right (positive value).

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$ \begin{align*} \vec{F}_{02} = F_{02,x} &= -1997.\;.\!.\!.\! {\rm \mu N} \\ \vec{F}_{03} = F_{03,y} &= -1123.\;.\!.\!.\! {\rm \mu N} \\ \end{align*} Since q_0 and q_2 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{02}$ points to the left (negative value).
Since q_0 and q_3 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{03}$ points downwards (negative value).

Embedded resources

Field lines of various extended charged objects