Block 12 - Capacitors and Capacitance

Student Group

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Learning objectives

After this 90-minute block, you can

1. define a **capacitor** and **capacitance** $C=\dfrac{Q}{U}$ and use $C=\varepsilon_0\,\varepsilon_{\rm r}\,\dfrac{A}{d}$ for an ideal plate capacitor, including unit checks $[C]={\rm F}$.
2. relate fields and material: $\vec{D}=\varepsilon\,\vec{E}$, $Q=\displaystyle\oint \vec{D}\cdot{\rm d}\vec{A}$ (Gauss), and $U=\displaystyle\int \vec{E}\cdot{\rm d}\vec{s}$.
3. compute C for key geometries (parallel plates, coaxial, spherical) and explain how A, d, $\varepsilon_{\rm r}$ scale C.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (8 min):
    1. Quick recall quiz: $C=\dfrac{Q}{U}$, $\vec{D}=\varepsilon\vec{E}$, units of E (${\rm V/m}$), D (${\rm C/m^2}$).
    2. Estimate: how C changes when A doubles or d halves (plate model).
2. Core concepts & derivations (60 min):
    1. From fields to C (plate capacitor): $U=\int\vec{E}\cdot{\rm d}\vec{s}=E\,d$, $Q=\oint\vec{D}\cdot{\rm d}\vec{A}=D\,A$, $\Rightarrow C=\varepsilon_0\varepsilon_{\rm r}\dfrac{A}{d}$. Dimensional check: $[\varepsilon_0\varepsilon_{\rm r}A/d]={\rm F}$.
    2. Other geometries: coaxial and spherical capacitor formulas; where fields are highest (edge intuition kept qualitative).
3. Practice (20 min):
    1. Mini-calcs: (i) C of given $A,d,\varepsilon_{\rm r}$; (ii) coaxial C per length; (iii) allowable U from E_0 and d; (iv) energy at given U.
    2. Discuss the provided “glass plate in capacitor” task.
4. Wrap-up (2 min): Summary box + pitfalls checklist; connect to next block (capacitor circuits).

Conceptual overview

1. A **capacitor** is two conductors separated by a dielectric. It stores **charge** and **energy** in the electric field; no conduction current flows through the ideal dielectric. :contentReference[oaicite:15]{index=15}
2. **Capacitance** measures how much charge per volt: $C=\dfrac{Q}{U}$. For parallel plates, $C=\varepsilon_0\varepsilon_{\rm r}\dfrac{A}{d}$ → increase A or $\varepsilon_{\rm r}$, decrease d to raise C. :contentReference[oaicite:16]{index=16}
3. **Other geometries:** Closed forms exist for coaxial and spherical capacitors; useful as building blocks and for cables/sensors. :contentReference[oaicite:18]{index=18}

Core content

Capacitor

- A capacitor can “store” charges. The total charge of a two-plate capacitor is in general 0.
- From the mechanical point of view a capacitor has two electrodes (= conductive areas), which are separated by a dielectric (= non-conductor).
- In a capacitor an electric field can be established without charge carriers moving internally.
- The characteristic of the capacitor is the capacitance C.
- In addition to the capacitance, every capacitor also has resistance and inductance. However, both of these are usually very small and are neglected for an ideal capacitor.
- Examples of capacitor are
  - the electrical component “capacitor”,
  - an open switch,
  - a wire related to ground,
  - a human being

$\rightarrow$ Thus, for any arrangement of two conductors separated by an insulating material, a capacitance can be specified.

Capacitance C

The capacitance C can be derived as follows:

1. A plate capacitor has a nearly homogenious field.
   Therefore, it is given for the voltage:
   $$U = \int \vec{E} {\rm d} \vec{s} = E \cdot l$$
   and hence
   $$E= {{U}\over{l}}$$ or with $D = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot E $ $$D= \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot {{U}\over{l}}$$
2. Furthermore, the charge Q can be given as $${Q= \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \; \vec{D} \cdot {\rm d} \vec{A} $$ The idealizion for the plate capacitor leads to: $$Q=D \cdot A$$.
3. Thus, the charge Q is given by: \begin{align*} Q = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot {{U}\over{l}} \cdot A \end{align*}

4. This means that $Q \sim U$, given the geometry (i.e., A and d) and the dielectric ($\varepsilon_{ \rm r} $).
5. So it is reasonable to determine a proportionality factor ${{Q}\over{U}}$.

The capacitance C of an idealized plate capacitor is defined as

\begin{align*} \boxed{C = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot {{A}\over{l}} = {{Q}\over{U}}} \end{align*}

Some of the main results here are:

- The capacity can be increased by increasing the dielectric constant $\varepsilon_{ \rm r} $, given the the same geometry.
- As near together the plates are as higher the capacity will be.
- As larger the area as higher the capacity will be.

This relationship can be examined in more detail in the following simulation:

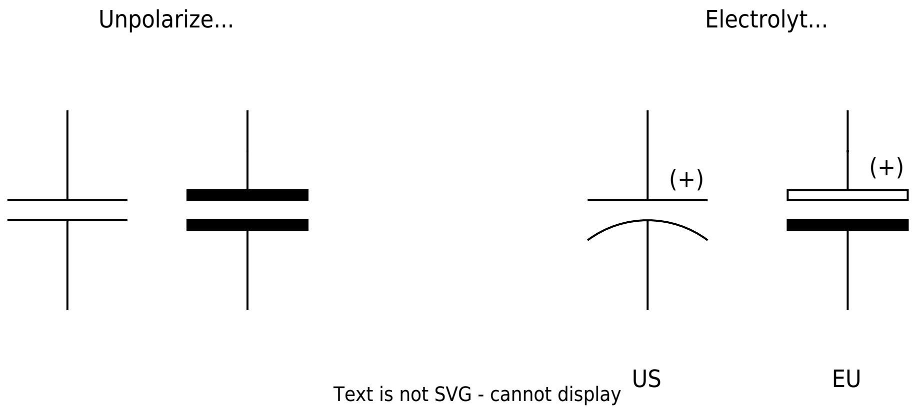
Capacitor lab
    If the simulation is not displayed optimally, this link can be used.

The figure 1 shows the topology of the electric field inside a plate capacitor.

Fig. 1: Topological situation inside a plate capacitor

Symbols

- The symbol of a general capacitor is given be two parallel lines nearby each other.
- Since **electrolytic capacitors** can only withstand voltage in one direction, the **polarisation** is often shown by a curved electrode (US) or a unfilled one (EU).

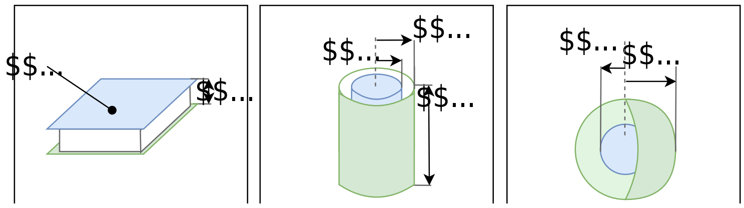


Designs and types of capacitors

To calculate the capacitance of different designs, the definition equations of $\vec{D}$ and $\vec{E}$ are used. This can be viewed in detail, e.g., in this video.

Based on the geometry, different equations result (see also figure 2).

Fig. 2: geometry of capacitors



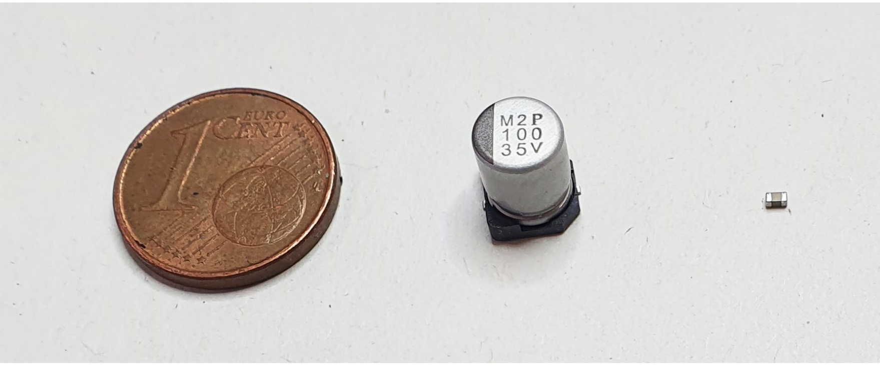
Shape of the Capacitor	**Parameter**	**Equation for the Capacity**
plate capacitor	area A of plate distance l between plates	$C = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot {{A}\over{l}}$
cylinder capacitor	radius of outer conductor $R_{ \rm o}$ radius of inner conductor $R_{ \rm i}$ length l	$C = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot 2\pi {{l}\over{{\rm ln} \left({{R_{ \rm o}}\over{R_{ \rm i}}}\right)}}$
spherical capacitor	radius of outer spherical conductor $R_{ \rm o}$ radius of inner spherical conductor $R_{ \rm i}$	$C = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot 4 \pi {{R_{ \rm i} \cdot R_{ \rm o}}\over{R_{ \rm o} - R_{ \rm i}}}$

Fig. 3: Structural shapes of Capacitors

In figure 3 different designs of capacitors can be seen:

1. **rotary variable capacitor** (also variable capacitor or trim capacitor).
    1. A variable capacitor consists of two sets of plates: a fixed set and a movable set (stator and rotor). These represent the two electrodes.
    2. The movable set can be rotated radially into the fixed set. This covers a certain area of A.
    3. The size of the area is increased by the number of plates. Nevertheless, only small capacities are possible because of the necessary distance.
    4. Air is usually used as the dielectric; occasionally, small plastic or ceramic plates are used to increase the dielectric constant.
2. **multilayer capacitor**
    1. In the multilayer capacitor, there are again two electrodes. Here, too, the area A (and thus the capacitance C) is multiplied by the finger-shaped interlocking.
    2. Ceramic is used here as the dielectric.
    3. The multilayer ceramic capacitor is also referred to as KerKo or MLCC.

    4. The variant shown in (2) is an SMD variant (surface mound device).
3. Disk capacitor
    1. A ceramic is also used as a dielectric for the disk capacitor. This is positioned as a round disc between two electrodes.
    2. Disc capacitors are designed for higher voltages, but have a low capacitance (in the microfarad range).
4. **Electrolytic capacitor**, in German also referred to as *Elko* for *Elektrolytkondensator*
    1. In electrolytic capacitors, the dielectric is an oxide layer formed on the metallic electrode. The second electrode is the liquid or solid electrolyte.
    2. Different metals can be used as the oxidized electrode, e.g., aluminum, tantalum, or niobium.
    3. Because the oxide layer is very thin, a very high capacitance results (depending on the size: up to a few millifarads).
    4. Important for the application is that it is a polarized capacitor. I.e., it may only be operated in one direction with DC voltage. Otherwise, a current can flow through the capacitor, which destroys it and is usually accompanied by an explosive expansion of the electrolyte. To avoid reverse polarity, the negative pole is marked with a dash.
    5. The electrolytic capacitor is built up wrapped, and often has a cross-shaped predetermined breaking point at the top for gas leakage.
5. **film capacitor**, in German also referred to as *Folko*, for *Folienkondensator*.
    1. A material similar to a “chip bag” is used as an insulator: a plastic film with a thin, metalized layer.
    2. The construction shows a high pulse load capacitance and low internal ohmic losses.
    3. In the event of an electrical breakdown, the foil enables “self-healing”: the metal coating evaporates locally around the breakdown. Thus the short-circuit is canceled again.
    4. With some manufacturers, this type is referred to as MKS (Mmetallized foilccapacitor, Polyester).
6. **Supercapacitor** (engl. Super-Caps)
    1. As a dielectric is - similar to the electrolytic capacitor - very thin. In the actual sense, there is no dielectric at all.
    2. The charges are not only stored in the electrode, but - similar to a battery - the charges are transferred into the electrolyte. Due to the polarization of the charges, they surround themselves with a thin (atomic) electrolyte layer. The charges then accumulate at the other electrode.
    3. Supercapacitors can achieve very large capacitance values (up to the Kilofarad range), but only have a low maximum voltage



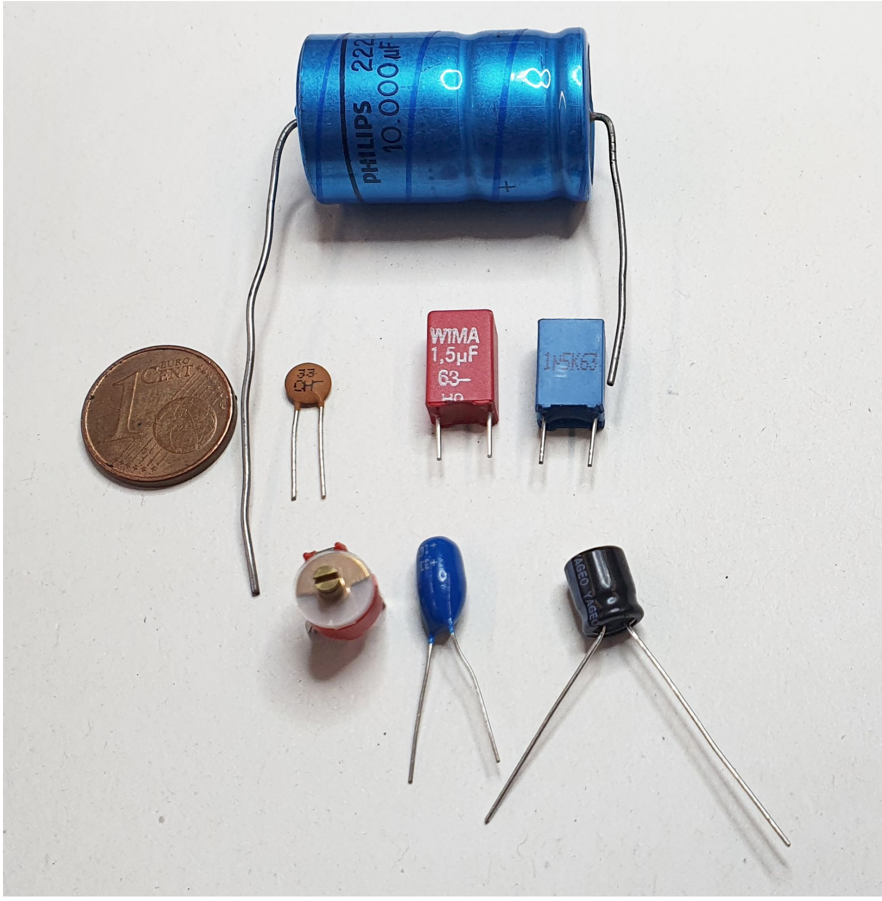


Fig. 2: types of capacitors

In figure 2 are shown different capacitors:

1. The above two SMD capacitors
    1. On the left a $100~{ \rm µF}$ electrolytic capacitor
    2. On the right a $100~{ \rm nF}$ MLCC in the commonly used Surface-mount_technology 0603 ($1.6~{ \rm mm}$ x $0.8~{ \rm mm}$)
2. below different THT capacitors (Through Hole Technology)
    1. A big electrolytic capacitor with $10~{ \rm mF}$ in blue, the positive terminal is marked with $+$
    2. In the second row is a Kerko with $33~{ \rm pF}$ and two Folkos with $1,5~{ \rm µF}$ each
    3. In the bottom row, you can see a trim capacitor with about $30~{ \rm pF}$ and a tantalum electrolytic capacitor and another electrolytic capacitor

Various conventions have been established for designating the capacitance value of a capacitor various conventions.

Note:

1. There are polarized capacitors. With these, the installation direction and current flow must be observed, as otherwise, an explosion can occur.
2. Depending on the application - and the required size, dielectric strength, and capacitance - different types of capacitors are used.
3. The calculation of the capacitance is usually <u>not</u> via $C = \varepsilon_0 \cdot \varepsilon_{ \rm r} \cdot {{A}\over{l}} $. The capacitance value is given.

4. The capacitance value often varies by more than $\pm 10~{ \rm \%}$. I.e., a calculation accurate to several decimal places is rarely necessary/possible.

1. The charge current seems to be able to flow through the capacitor because the charges added to one side induce correspondingly opposite charges on the other side.

Common pitfalls

- **Mixing up geometry symbols.** Use d (or l) strictly as **plate spacing** and A as **active area** in $C=\varepsilon_0\varepsilon_{\rm r}\dfrac{A}{d}$. Check units to catch mistakes.
- **Forgetting the field relations.** $U=\int \vec{E}\cdot{\rm d}\vec{s}$ and $Q=\oint \vec{D}\cdot{\rm d}\vec{A}$; without them, layered-dielectric problems are guessed instead of solved.
- **Assuming conduction through the dielectric.** The apparent "current through a capacitor" is displacement-related; no charge carriers traverse the ideal dielectric.
- **Real-part issues.** Ignoring polarity of electrolytics and tolerance spreads ($\pm 10~\%$ and more) causes design errors; pick suitable component types.

Exercises

Task 5.5.1 induced Charges

A plate capacitor with a distance of $d = 2 ~{ \rm cm}$ between the plates and with air as dielectric ($\varepsilon_{ \rm r}=1$) gets charged up to $U = 5~{ \rm kV}$. In between the plates, a thin metal foil with the area $A = 45~{ \rm cm^2}$ is introduced parallel to the plates.

Calculate the amount of the displaced charges in the thin metal foil.

Tips for the solution

- What is the strength of the electric field E in the capacitor?
- Calculate the displacement flux density D
- How can the charge Q be derived from D?

Result

$Q = 10 ~{ \rm nC}$

Task 5.5.2 Manipulating a Capacitor I

An ideal plate capacitor with a distance of $d_0 = 7 ~{ \rm mm}$ between the plates gets charged up to $U_0 = 190~{ \rm V}$ by an external source. The source gets disconnected. After this, the distance between the plates gets enlarged to $d_1 = 7 ~{ \rm cm}$.

1. What happens to the electric field and the voltage?

2. How does the situation change (electric field/voltage), when the source is not disconnected?

Tips for the solution

- Consider the displacement flux through a surface around a plate

Result

1. $U_1 = 1.9~{ \rm kV}$, $E_1 = 27~{ \rm kV/m}$
2. $U_1 = 190~{ \rm V}$, $E_1 = 2.7~{ \rm kV/m}$

Task 5.5.3 Manipulating a Capacitor II

An ideal plate capacitor with a distance of $d_0 = 6~{ \rm mm}$ between the plates and with air as dielectric ($\varepsilon_{\rm r}=1$) is charged to a voltage of $U_0 = 5~{ \rm kV}$. The source remains connected to the capacitor. In the air gap between the plates, a glass plate with $d_{ \rm g} = 4 ~{ \rm mm}$ and $\varepsilon_{ \rm r} = 8$ is introduced parallel to the capacitor plates.

1. Calculate the partial voltages on the glas $U_{ \rm g}$ and on the air gap $U_{ \rm a}$.

Tipps

- Build a formula for the sum of the voltages first
- How is the voltage related to the electric field of a capacitor?

Path

The sum of the voltages across the glass and the air gap gives the total voltage U_0, and each individual voltage is given by the E-field in the individual material by $E = {{U}\over{d}}$: $$\begin{align*} U_0 &= U_{\rm g} + U_{\rm a} \\ &= E_{\rm g} \cdot d_{\rm g} + E_{\rm a} \cdot d_{\rm a} \end{align*}$$

The displacement field D must be continuous across the different materials since it is only based on the charge Q on the plates. $$\begin{align*} D_{\rm g} &= D_{\rm a} \\ \varepsilon_0 \varepsilon_{\rm r, g} \cdot E_{\rm g} &= \varepsilon_0 \cdot E_{\rm a} \end{align*}$$

Therefore, we can put $E_\rm a= \varepsilon_{\rm r, g} \cdot E_\rm g $ into the formula of the total voltage and rearrange to get $E_\rm g$: $$\begin{align*} U_0 &= E_{\rm g} \cdot d_{\rm g} + \varepsilon_{\rm r, g} \cdot E_{\rm g} \cdot d_{\rm a} \\ &= E_{\rm g} \cdot (d_{\rm g} + \varepsilon_{\rm r, g} \cdot d_{\rm a}) \\ \rightarrow E_{\rm g} &= {{U_0}\over{d_{\rm g} + \varepsilon_{\rm r, g} \cdot d_{\rm a}}} \end{align*}$$

Since we know that the distance of the air gap is $d_{\rm a} = d_0 - d_{\rm g}$ we can calculate: $$\begin{align*} E_{\rm g} &= {{5'000 ~{\rm V}}\over{0.004 ~{\rm m} + 8 \cdot 0.002 ~{\rm m}}} \\ &= 250 ~\rm{{kV}\over{m}} \end{align*}$$

By this, the individual voltages can be calculated: $$\begin{align*} U_{ \rm g} &= E_{\rm g} \cdot d_\rm g &= 250 ~\rm{{kV}\over{m}} \cdot 0.004 ~\rm m &= 1 ~{\rm kV}\\ U_{ \rm a} &= U_0 - U_{ \rm g} &= 5 ~{\rm kV} - 1 ~{\rm kV} &= 4 ~{\rm kV}\\ \end{align*}$$

Result

$U_{ \rm a} = 4 ~{ \rm kV}$, $U_{ \rm g} = 1 ~{ \rm kV}$

2. What would be the maximum allowed thickness of a glass plate, when the electric field in the air gap shall not exceed $E_{0,\rm max}=12~{\rm kV/cm}$?

Path

Again, we can start with the sum of the voltages across the glass and the air gap, and use the formula from the displacement field $D = \varepsilon_0 \varepsilon_{\rm r, g} \cdot E_{\rm g} = \varepsilon_0 \cdot E_\rm a $.

Now we shall eliminate $E_\rm g$, since $E_\rm a$ is given in the question. $$\begin{align*} U_0 &= E_{\rm g} \cdot d_{\rm g} + E_{\rm a} \cdot d_{\rm a} \\ &= {{E_\rm a}\over{\varepsilon_{\rm r,g}}} \cdot d_{\rm g} + E_{\rm a} \cdot d_{\rm a} \end{align*}$$

The distance $d_\rm a$ for the air is given by the overall distance d_0 and the distance for glass $d_\rm g$: $$\begin{align*} d_{\rm a} = d_0 - d_{\rm g} \end{align*}$$

This results in: $$\begin{align*} U_0 &= {{E_{\rm a}}\over{\varepsilon_{\rm r,g}}} \cdot d_{\rm g} + E_{\rm a} \cdot (d_0 - d_{\rm g}) \\ \rightarrow {{U_0}\over{E_{\rm a}}} &= {{1}\over{\varepsilon_{\rm r,g}}} \cdot d_{\rm g} + d_0 - d_{\rm g} \\ &= d_{\rm g} \cdot ({{1}\over{\varepsilon_{\rm r,g}}} - 1) + d_0 \\ d_{\rm g} &= { { {{U_0}\over{E_{\rm a}}} - d_0 } \over { {{1}\over{\varepsilon_{\rm r,g}}} - 1 } } \\ &= { { d_0 - {{U_0}\over{E_{\rm a}}} } \over { 1 - {{1}\over{\varepsilon_{\rm r,g}}} } } \end{align*}$$

With the given values: $$\begin{align*} d_{\rm g} &= { { 0.006 ~\rm m - {{5 ~\rm kV}\over{12 ~\rm kV/cm}} } \over { 1 - {{1}\over{8}} } } &= {{8}\over{7}} \left(0.006 - {{5}\over{1200}} \right) ~\rm m \end{align*}$$

Result

$d_{ \rm g} = 2.10 ~{\rm mm}$

Task 5.5.4 Spherical capacitor

Two concentric spherical conducting plates set up a spherical capacitor. The radius of the inner sphere is $r_{ \rm i} = 3~{ \rm mm}$, and the inner radius from the outer sphere is $r_{ \rm o} = 9~{ \rm mm}$.

1. What is the capacity of this capacitor, given that air is used as a dielectric?
2. What would be the limit value of the capacity when the inner radius of the outer sphere goes to infinity ($r_{ \rm o} \rightarrow \infty$)?

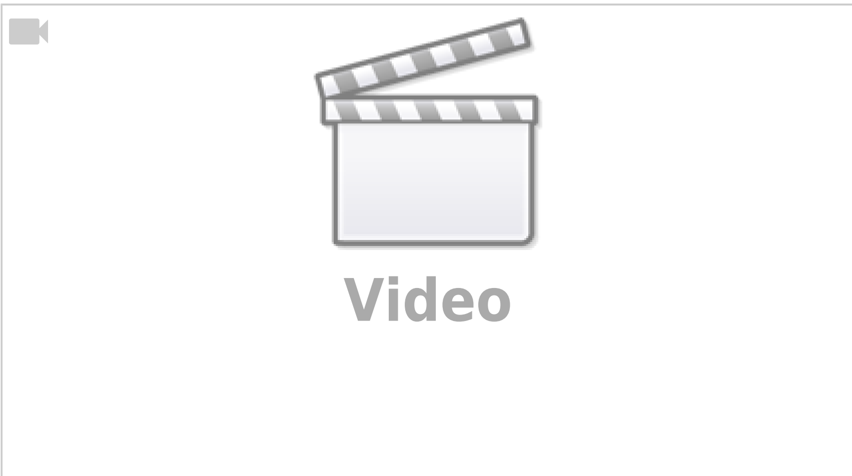
Tips for the solution

- What is the displacement flux density of the inner sphere?
- Out of this derive the strength of the electric field E
- What ist the general relationship between U and $\vec{E}$? Derive from this the voltage between the spheres.

Result

1. $C = 0.5~{ \rm pF}$
2. $C_{\infty} = 0.33~{ \rm pF}$

Task 5.5.5 Applying Gauss's law: Electric Field of a line charge

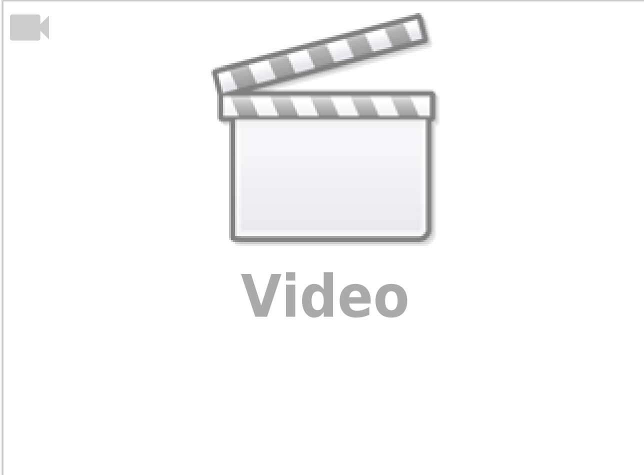


Embedded resources

The background behind the dielectric constant $\varepsilon_{ \rm r} $ and the field is explained in the following video

Electrolytic capacitors can explode!


Video


Video