Block 14 - The steady Conduction Field

Student Group

First Name	Surname	Matrikel Nr.

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Learning objectives

After this 90-minute block, you can

- explain what a **steady (stationary) conduction field** is and relate it to the electrostatic field (cause/effect view: $\vec{E}$ vs. $\vec{D}$; conduction uses $\vec{E}$ and material σ).
- use the **current-density law** $\vec{j}=\sigma\,\vec{E}$ and the **current flux** $I=\iint_A \vec{j}\cdot{\rm d}\vec{A}$ with correct surface orientation.
- derive and calculate **conductance** G and **resistance** R for key geometries (parallel plates , coaxial conductor).

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

In the discussion of the electrostatic field in principle, no charges in motion were considered.
This lead to multiple fomulas, which are aggregated in the following diagram:

Fig. 1: summary of electro static field

One outcome was, that the capacitance is defined as:

\begin{align*} C &= {{Q}\over{U}} \\ &= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{D} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} \end{align*}

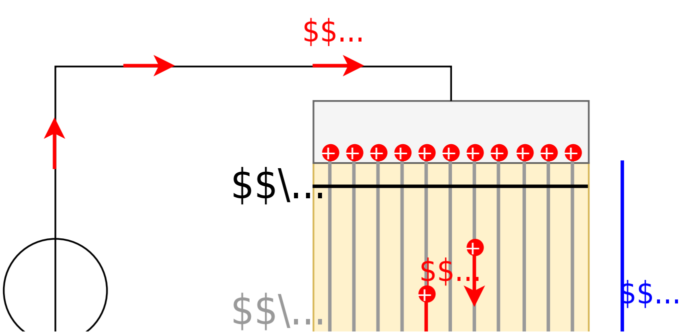
Now the motion of charges shall be considered explicitly.
With the knowledge of th electrostatic field, we want to see, whether we can calulate the resistance of more complicated geometries.

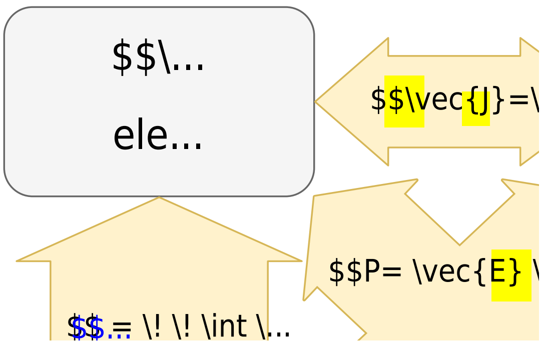
For this we want to introduce the current density J: The current density here describes how charge carriers move together (collectively).
The stationary current density describes the charge carrier movement if a **direct voltage** is the cause of the movement.
Then, a constant direct current flows in the stationary electric flow field. Thus, there is no time dependency on the current:

$\large{{{\rm d}I}\over{{\rm d}t}}=0$

Important: Up to now it was considered, that charges had moved through a field in the past or could be moved in the future. Now, the exact moment of moving the charge is considered.

Fig. 2: summary of conduction field





By comparison, we see now, that the resistance can be defined as:

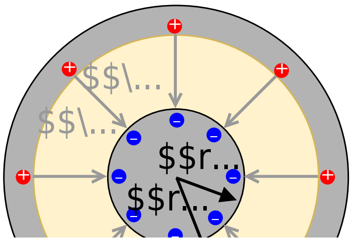
\begin{align*} {{1}\over{R}} &= {{I}\over{U}} \\ &= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{J} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} \end{align*}

Given the results from block 11 we can derive:

Fig. 3: current between parallel plates

- for a current between **parallel plates**
  - The current density is given as: \begin{align*} J = {{I}\over{A}} = \sigma \cdot E = {\rm const.} \end{align*}
  - This leads to the electric field: \begin{align*} E = {{J}\over{\sigma}} \end{align*}
  - The resistance value is given as: \begin{align*} {{1}\over{R}}&= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{J} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} = {{J} \cdot \rlap{\int_A}\int \; {\rm d} {A}\over{{E} \cdot \int \,{\rm d} {s} }} \end{align*}\begin{align*} \boxed{ {{1}\over{R}}= {{\sigma A}\over{l}} }_\text{between parallel plates} \end{align*}

Fig. 4: current between coaxial plates



- for a current between **coaxial plates**
  - The current density is given as: \begin{align*} J = {{I}\over{2\pi \cdot l \cdot r}} \end{align*}
  - The resistance value is given as: \begin{align*} \boxed{ {{1}\over{R}}=\dfrac{2\pi\sigma l}{\ln(r_a/r_i)} }_\text{between coaxial plates}\end{align*}

Common pitfalls

- ...

Exercises

Worked examples

...

Embedded resources

Explanation (video): ...

The online book 'University Physics II' is strongly recommended as a reference for this chapter. Especially the following chapters:

- Chapter 9.3 Model of Conduction in Metals