

Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Student Group

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Block 16 - Ampère's Law and Magnetomotive Force (MMF)

16.0 Intro

16.0.1 Learning objectives

After this 90-minute block, you can

- ...

16.0.2 Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

16.0.3 90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

16.0.4 Conceptual overview

1. ...

16.1 Core content

16.1.1 Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem of a single wire was considered in formula. I.e a current I and the length s of a magnetic field line around the wire was given to calculate the magnetic field strength H :

$$\begin{aligned} \quad H_{\varphi} = \frac{I}{2 \pi r} &\quad \Leftrightarrow \\ \quad I = H_{\varphi} \cdot s &\quad \text{\textit{applies only to the long, straight conductor}} \end{aligned}$$

Now, this shall be generalized. For this purpose, we will look back at the electric field. For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

$$U = E \cdot s \quad \text{\textit{applies to plate capacitor only}}$$

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.

This was extended to the voltage between to points 1 and 2 . Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is "0".

$$U_{12} = \int_1^2 \vec{E} \cdot d\vec{s} \quad \text{\textit{applies to rotational symmetric problems only}} \\ \oint \vec{E} \cdot d\vec{s} = 0$$

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.

Because of the similarity the so-called **magnetic potential difference V_m** between point 1 and 2 is introduced:

$$V_m = \int_1^2 H \cdot ds \quad \text{\textit{applies to rotational symmetric problems only}}$$

$$\boxed{V_m = V_{m, 12} = \int_1^2 \vec{H} \cdot d\vec{s}} \quad \text{\textit{applies to rotational symmetric problems only}}$$

We need to take a closer look here. Any closed path in the static electric field leads to a potential difference of $U = \oint \vec{E} \cdot d\vec{s} = 0$.

BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily** 0 ! $V_m = \oint \vec{H} \cdot d\vec{s} = \theta$

Another new quantity is introduced: the **magnetic voltage θ** :

1. The magnetic voltage θ is the magnetic potential difference on a closed path.
2. Since the magnetic voltage θ is valid for exactly one turn along our single wire, θ is also equal to the current through the wire:

$$\theta = H \cdot s = I \quad \text{\textit{applies only to the long, straight conductor}}$$

3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to I .
4. The magnetic voltage is generalized in the following box.

Notice:

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

$\boxed{\oint_{\mathcal{S}} \vec{H} \cdot d\vec{s} = \theta}$	<p>The magnetic voltage θ can be given as</p> <ul style="list-style-type: none"> • $\theta = I$ for a single conductor • $\theta = N \cdot I$ for a coil • $\theta = \sum_n I_n$ for multiple conductors • $\theta = \int_A \vec{S} \cdot d\vec{A}$ for any spatial distribution (see block15)
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The unit of the magnetic voltage θ is **Ampere** (or **Ampere-turns**).

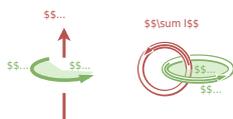
In the english literature the magnetic voltage is called **Magnetomotive force**

Notice:

$\oint \vec{s}$ and $\int \vec{A}$ in $\oint \vec{H} \cdot d\vec{s} = \theta = \int_A \vec{S} \cdot d\vec{A}$ build a right-hand system.

1. Once the thumb of the right hand is pointing along $\int \vec{A}$, the fingers of the right hand show the correct direction for $\oint \vec{s}$ for positive \vec{H} and \vec{S}
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



16.1.2 Recap of the fieldline images

longitudinal coil

Fig. 2: Magnetic field in a longitudinal coil

A longitudinal coil can be seen in [figure 2](#).

The created field density of the coil can be derived from Ampere's Circuital Law

$$\begin{aligned} \oint \vec{H}(t) \cdot d\vec{s} &= \int \vec{H}_{\text{inner}}(t) \cdot d\vec{s} + \int \vec{H}_{\text{outer}}(t) \cdot d\vec{s} \\ &= \int \vec{H}(t) \cdot d\vec{s} + 0 \\ &= \int \vec{H}(t) \cdot d\vec{s} \end{aligned}$$

The magnetic field in a toroidal coil is often considered as homogenous in the inner volume, when the length l is much larger than the diameter: $l \gg d$.

With a given number N of windings, the magnetic field strength H is

toroidal coil

Fig. 3: Magnetic field in a toroidal coil

A toroidal coil has a donut-like setup. This can be seen in [figure 3](#).

The toroidal coil is often defined by:

- The minor radius r : The radius of the circular cross-section of the coil.
- The major radius R : The distance from the center of the entire toroid (the center of the hole) to the center of the circular cross-section of the coil.

For reasons of symmetry, it shall get clear that the field lines form concentric circles. Also the magnetic field strength H in a toroidal coil is often considered as homogenous, when $R \gg r$. With a given number N of windings, the magnetic field strength H is

$$\theta = H \cdot l = N \cdot I$$

$$H = \frac{N \cdot I}{l} \quad \text{longitudinal coil}$$

$$\theta = H \cdot 2\pi R = N \cdot I$$

$$H = \frac{N \cdot I}{2\pi R} \quad \text{toroidal coil}$$

16.2 Common pitfalls

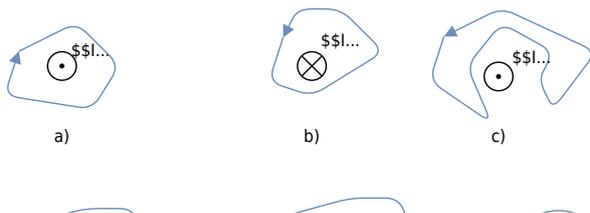
- ...

16.3 Exercises

Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors
 Result **e)**

e) $V_{(1\text{mm}, \theta)} = 20\text{V} = (2.5\text{A} - 4.5\text{A}) \cdot 4\text{m} = -2.5\text{A} \cdot 4\text{m} = -10\text{V}$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let $I_1 = 2\text{A}$ and $I_2 = 4.5\text{A}$ be valid.

In each case, the magnetic potential difference V_{m} along the drawn path is sought.

Path

- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

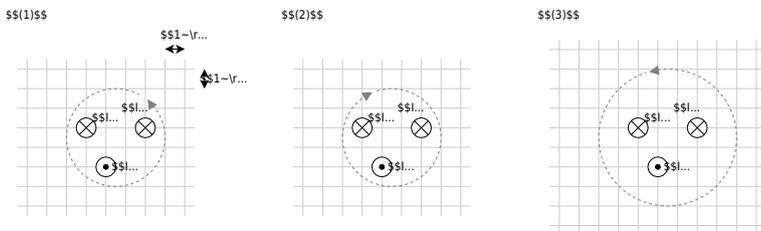
Exercise E11 Magnetic Voltage (written test, approx. 6 % of a 120-minute written test, SS2021)

The following images show cross-sections of electrical cables.

A closed path is shown as a dashed line. The magnetic voltage θ on these paths shall be analyzed.

The following values are given for the currents:

- $\begin{aligned} \theta_{(1)} &= -4 \text{ A} \\ \theta_{(2)} &= 0 \text{ A} \\ \theta_{(3)} &= 5 \text{ A} \\ \theta_{(4)} &= 5 \text{ A} \end{aligned}$
- $I_3 = 1 \text{ A}$
 - $I_4 = 4 \text{ A}$



Specify which magnetic voltages $\theta_{(1)}$, $\theta_{(2)}$, and $\theta_{(3)}$ result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $$I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A} - 5 \text{ A} - 1 \text{ A}$$
- $$I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$$
- $$I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$$

Exercise E1 Magnetic Potential

(written test, approx. 8 % of a 120-minute written test, SS2024)

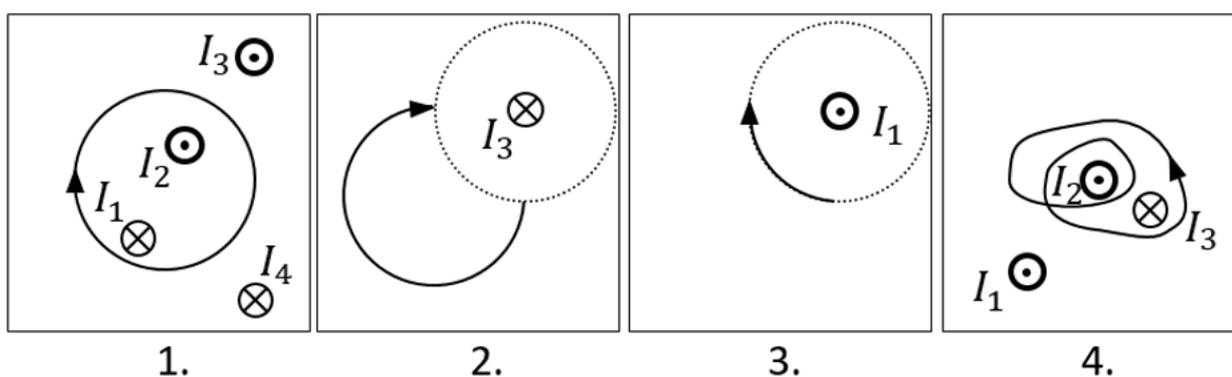
Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 \text{ A}$
- $|I_2|=5 \text{ A}$
- $|I_3|=11 \text{ A}$
- $|I_4|=7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task: $+I_1 - I_2 = -3 \text{ A}$
2. Task: $+\frac{1}{4} I_3 = 11/4 \text{ A}$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
3. Task: $-\frac{1}{4} I_1 = -0.5 \text{ A}$
4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ A}$

Exercise E9 Fields of an coax Cable
(written test, approx. 12 % of a 120-minute written test, SS2024)

2. With the graph of the magnitude of D (r) with parameters $D_0 = 0.6 \text{ nC/m}^2$ and $D_1 = 0.5 \text{ nC/m}^2$ as shown in the cross-section of the coax cable with $(0, 0)$ as center of the coax cable dimensions and labels for the diagram appears:

Path

- Inner conductor: $+3.3 \text{ nA}$, $+10 \text{ nC}$ (current into the plane of the path diagram)
- Outer conductor: -3.3 nA , 0 nC (current out of the plane of diagram)
- for $(0.1 \text{ mm} | 0)$: $E_{\text{I}} = 5.78 \text{ V/m}$
- for $(0.55 \text{ mm} | 0)$: $E_{\text{O}} = 6.985 \text{ V/m}$

The magnitude of the electric displacement field D can be calculated by: $\int D \cdot dA = Q_{\text{enc}}$.

Here, in any position r from the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

For the shell as a surface of the cylinder, the area is $A = 2\pi r \cdot l$. This leads to: $D(r) = \frac{Q_{\text{enc}}}{A} = \frac{Q_{\text{enc}}}{2\pi r \cdot l}$.

This is proportional to the area within this radius. Therefore, the formula $H = \frac{I_{\text{enc}}}{2\pi r}$ is used.

So, we get for $D_{\text{I}}(r)$ at $r = 0.1 \text{ mm}$, and $D_{\text{O}}(r)$ at $r = 0.55 \text{ mm}$.

For r within the outer conductor one also gets a linear proportionality with a similar approach: $D(r) = \frac{Q_{\text{enc}}}{2\pi r \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot \{0.1 \cdot 10^{-3} \text{ m} \cdot 0.5 \text{ m}\}} \cdot \frac{1}{r}$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive.

But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength H at $(0.1 \text{ mm} | 0)$ and $(0.55 \text{ mm} | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2\pi r}$

So, we get for H_{I} at $(0.1 \text{ mm} | 0)$, and H_{O} at $(0.55 \text{ mm} | 0)$

~mm | 0)\$:

$$\begin{aligned} H_{\text{i}} &= \frac{I}{2 \pi \cdot r_{\text{i}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{ 0.1 \cdot 10^{-3} \text{ m} \}} \\ H_{\text{o}} &= \frac{I}{2 \pi \cdot r_{\text{o}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{ 0.55 \cdot 10^{-3} \text{ m} \}} \end{aligned}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards.

Therefore, the sign of the H -field is negative.

But here, only the magnitude was questioned!

Embedded resources

Explanation (video): ...

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