

Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

- Block 16 - Ampère's Law and Magnetomotive Force (MMF)** 2
- Learning objectives* 2
- Preparation at Home* 2
- 90-minute plan* 2
- Conceptual overview* 2
- Core content* 2
- Generalization of the Magnetic Field Strength* 2
 - Notice: 3
 - Notice: 4
- Common pitfalls* 5
- Exercises* 5
 - Task 3.2.3 Magnetic Potential Difference 5
 - Exercise E1 Magnetic Voltage (written test, approx. 6 % of a 120-minute written test, SS2021) 7
 - Exercise E1 Magnetic Potential (written test, approx. 8 % of a 120-minute written test, SS2024) 8
 - Exercise E6 Fields of an coax Cable (written test, approx. 12 % of a 120-minute written test, SS2024) 9
- Embedded resources* 10

Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Learning objectives

After this 90-minute block, you can

- ...

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. ...

Core content

Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem on a single wire was considered in formula, when the current I and the length s of a magnetic field line around it is given:

$$\begin{aligned} \quad H_{\varphi} &= \frac{I}{s} = \frac{I}{2 \cdot \pi \cdot r} \quad \Leftrightarrow \end{aligned}$$

$$\oint \mathbf{H} \cdot d\mathbf{s} = I_{\text{enc}} \quad | \quad \text{applies only to the long, straight conductor}$$

Now, this shall be generalized. For this purpose, we will look back at the electric field.

For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

$$U = E \cdot s \quad | \quad \text{applies to capacitor only}$$

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.

This was extended to the voltage between two points 1 and 2 . Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is "0".

$$\oint \mathbf{E} \cdot d\mathbf{s} = 0$$

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.

Because of the similarity the so-called **magnetic potential difference V_m** between point 1 and 2 is introduced:

$$V_m = \int \mathbf{H} \cdot d\mathbf{s} \quad | \quad \text{applies to rotational symmetric problems only}$$

$$V_m = \int \mathbf{H} \cdot d\mathbf{s} = \theta$$

We need to take a closer look here. Any closed path in the static electric field leads to a potential difference of $\oint \mathbf{E} \cdot d\mathbf{s} = 0$.

BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily** 0 ! $V_m = \oint \mathbf{H} \cdot d\mathbf{s} = \theta$

Another new quantity is introduced: the **magnetic voltage θ** :

1. The magnetic voltage θ is the magnetic potential difference on a closed path.
2. Since the magnetic voltage θ is valid for exactly one turn along our single wire, θ is also equal to the current through the wire:

$$\theta = H \cdot s = I \quad | \quad \text{applies only to the long, straight conductor}$$
3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to I .
4. The magnetic voltage is generalized in the following box.

Notice:

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

$$\boxed{\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s} = \theta}$$

The magnetic voltage θ can be given as

- $\theta = I$ for a single conductor
- $\theta = N \cdot I$ for a coil
- $\theta = \sum_n I_n$ for multiple conductors
- $\theta = \int_A \vec{S} \cdot \mathrm{d}\vec{A}$ for any spatial distribution (see [block15](#))

The unit of the magnetic voltage θ is **Ampere** (or **Ampere-turns**).

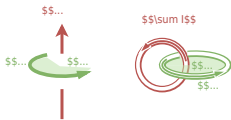
In the english literature the magnetic voltage is called **Magnetomotive force**

Notice:

$\oint_{\mathcal{S}} \vec{S} \cdot \mathrm{d}\vec{A}$ and $\int_A \vec{S} \cdot \mathrm{d}\vec{A}$ in $\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s} = \theta = \int_A \vec{S} \cdot \mathrm{d}\vec{A}$ build a right-hand system.

1. Once the thumb of the right hand is pointing along $\int_A \vec{S} \cdot \mathrm{d}\vec{A}$, the fingers of the right hand show the correct direction for $\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s}$ for positive $\int_A \vec{S} \cdot \mathrm{d}\vec{A}$
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



Common pitfalls

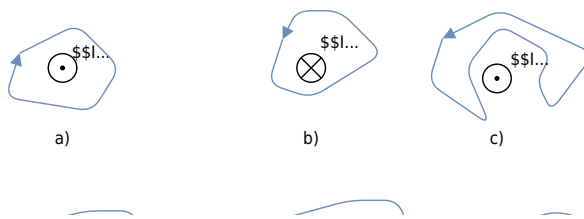
- ...

Exercises

Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors
 Result $\oint \mathbf{H} \cdot d\mathbf{l} = 2.5 \text{ A}$

$$\oint \mathbf{H} \cdot d\mathbf{l} = 2.5 \text{ A} = 2.5 \text{ A}$$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let $I_1 = 2 \text{ A}$ and $I_2 = 4.5 \text{ A}$ be valid.

In each case, the magnetic potential difference V_{m} along the drawn path is sought.

Path

- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

Exercise E1 Magnetic Voltage
(written test, approx. 6 % of a 120-minute written test, SS2021)

The following images show cross-sections of electrical cables.

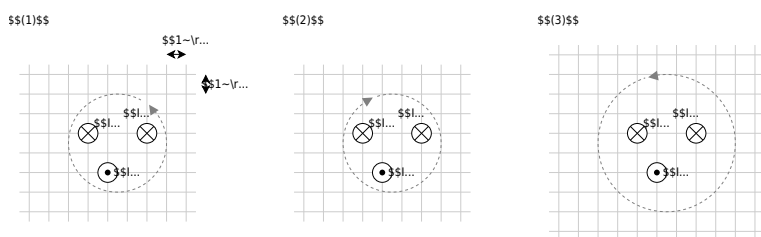
A closed path is shown as a dashed line. The magnetic voltage θ on these paths shall be analyzed.

The following values are given for the currents:

$I_1 = 5 \text{ A}$ $I_2 = 5 \text{ A}$ $I_3 = 1 \text{ A}$ $I_4 = 4 \text{ A}$

$\theta_{(1)} = -4 \text{ A}$ $\theta_{(2)} = 0 \text{ A}$

- $I_3 = 1 \text{ A}$
- $I_4 = 4 \text{ A}$



Specify which magnetic voltages $\theta_{(1)}$, $\theta_{(2)}$, and $\theta_{(3)}$ result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A} - 5$

- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$

Exercise E1 Magnetic Potential

(written test, approx. 8 % of a 120-minute written test, SS2024)

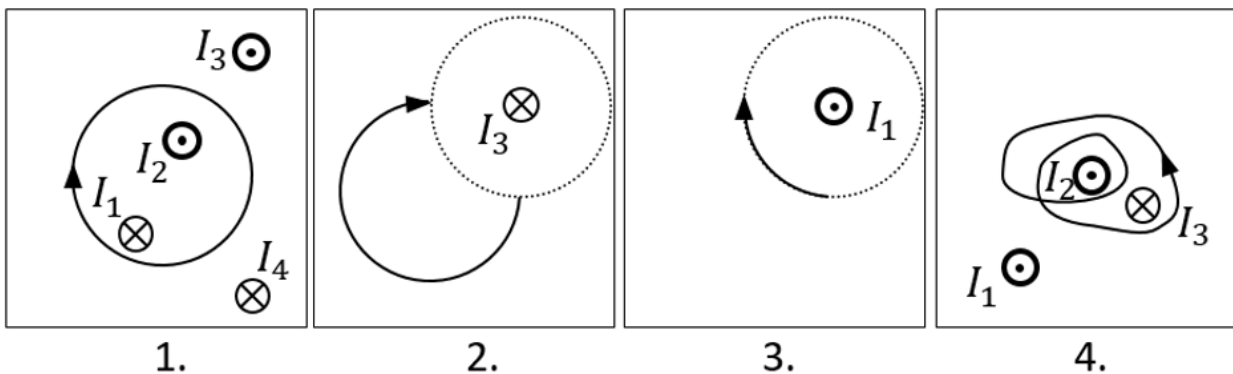
Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines.

Dotted lines are only there for symmetry aspects!

The wires conduct the following currents:

- $|I_1| = 2 \text{ A}$
- $|I_2| = 5 \text{ A}$
- $|I_3| = 11 \text{ A}$
- $|I_4| = 7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task: $+I_1 - I_2 = -3 \text{ A}$
2. Task: $+\frac{1}{4} I_3 = 11/4 \text{ A}$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)

3. Task: $I_1 = -0.5 \text{ A}$

4. Task: $2 I_2 - I_3 = -1 \text{ A}$

Exercise E6 Fields of an coax Cable

(written test, approx. 12 % of a 120-minute written test, SS2024)

2. With the graph of the magnitude of D (in C/m^2) with parameters $D_0 = 10^{-9} \text{ C/m}^2$ and $r_0 = 0.1 \text{ mm}$, the cross-section of the coaxial cable with $(0,6 \text{ mm} | 0)$ is centered in the Cartesian coordinate system and labeled for the diagram appears:

Path

- Inner conductor: $+3.3 \text{ mA}$, $+10 \text{ nC}$ (current into the plane of the diagram)
 - for $(0.1 \text{ mm} | 0)$: $B_i = 5.28 \dots \text{ A/m}^2$
- Outer conductor: -3.3 mA , 0 nC (current out of the plane of diagram)
 - for $(0.55 \text{ mm} | 0)$: $B_o = 6.985 \dots \text{ A/m}^2$

The magnitude of the electric displacement field D can be calculated by: $\int D \cdot dA = Q_{enc}$.

In general, the D -field is proportional to $\frac{1}{r}$ for the situation between both conductors.

For the situation on the surface of the inner conductor, the field D is directed with the conductor. within a circle with the radius x .

This is proportional to the area within this radius. Therefore, the formula $H = \frac{I}{2\pi r}$ gets $H(x) = \frac{I_i}{2\pi x}$ and $D(x) = \frac{Q}{2\pi x}$ at $x = 0.1 \text{ mm}$.

So, we get for D at $x = 0.1 \text{ mm}$ and D at $x = 0.55 \text{ mm}$: $D(0.1) = \frac{10^{-9} \text{ C}}{2\pi (0.1 \cdot 10^{-3} \text{ m})^2}$. This leads to a formula proportional to x .

For x within the outer conductor one also gets a linear proportionality with a $\frac{1}{r}$ approach: $D(r) = \frac{Q}{2\pi r}$ and $D(0.55) = \frac{10^{-9} \text{ C}}{2\pi (0.55 \cdot 10^{-3} \text{ m})^2}$.

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive. But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength H at $(0.1 \text{ mm} | 0)$ and $(0.55 \text{ mm} | 0)$?

0.55 ~mm | 0)?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2 \pi \cdot r}$

So, we get for H_{i} at $r_{\text{i}} = 0.1 \text{ ~mm}$, and H_{o} at $r_{\text{o}} = 0.55 \text{ ~mm}$:

$$\begin{aligned} H_{\text{i}} &= \frac{I}{2 \pi \cdot r_{\text{i}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ ~m}\}} \quad H_{\text{o}} &= \frac{I}{2 \pi \cdot r_{\text{o}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ ~m}\}} \end{aligned}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards.

Therefore, the sign of the H -field is negative.

But here, only the magnitude was questioned!

Embedded resources

Explanation (video): ...

From:

<https://wiki.mexle.org/> - MEXLE Wiki

Permanent link:

https://wiki.mexle.org/electrical_engineering_and_electronics_1/block16?rev=1763860090

Last update: **2025/11/23 02:08**

