

# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Student Group

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# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Learning objectives

After this 90-minute block, you can

- ...

## Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

## 90-minute plan

1. Warm-up (x min):
  1. ....
2. Core concepts & derivations (x min):
  1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

## Conceptual overview

1. ...

## Core content

### Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem of a single wire was considered in formula. I.e a current  $I$  and the length  $l$  of a magnetic field line around the wire was given to calculate the magnetic field strength  $H$ :

$$\begin{aligned} \quad H_{\varphi} = \frac{I}{s} = \frac{I}{2 \cdot \pi \cdot r} \quad \Leftrightarrow \\ \quad I = H_{\varphi} \cdot s \quad \quad \quad | \quad \text{applies only to the long, straight conductor} \end{aligned}$$

Now, this shall be generalized. For this purpose, we will look back at the electric field. For the electric field strength  $E$  of a capacitor with two plates at a distance of  $s$  and the potential difference  $U$  holds:

$$\begin{aligned} U = E \cdot s \quad \quad | \quad \text{applies to plate capacitor only} \end{aligned}$$

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.

This was extended to the voltage between to points  $1$  and  $2$ . Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is "0".

$$\begin{aligned} U_{12} &= \int_1^2 \vec{E} \cdot d\vec{s} \quad \quad U = \oint \vec{E} \cdot d\vec{s} = 0 \end{aligned}$$

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.

Because of the similarity the so-called **magnetic potential difference  $V_m$**  between point  $1$  and  $2$  is introduced:

$$\begin{aligned} V_m &= H \cdot s \quad \quad | \quad \text{applies to rotational symmetric problems only} \end{aligned}$$

$$\boxed{V_m = V_{m, 12} = \int_1^2 \vec{H} \cdot d\vec{s} \quad \quad V_m = \oint \vec{H} \cdot d\vec{s} = \theta}$$

We need to take a closer look here. Any closed path in the static electric field leads to a potential difference of  $U = \oint \vec{E} \cdot d\vec{s} = 0$ .

BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily**  $0$ !  $V_m = \oint \vec{H} \cdot d\vec{s} = \theta$

Another new quantity is introduced: the **magnetic voltage  $\theta$** :

1. The magnetic voltage  $\theta$  is the magnetic potential difference on a closed path.
2. Since the magnetic voltage  $\theta$  is valid for exactly one turn along our single wire,  $\theta$  is also equal to the current through the wire:

$$\begin{aligned} \theta = H \cdot s = I \quad \quad | \quad \text{applies only to the long, straight conductor} \end{aligned}$$

3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to  $I$ .
4. The magnetic voltage is generalized in the following box.

**Notice:**

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage  $\theta$  (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

$\boxed{\oint_{\mathcal{S}} \vec{H} \cdot d\vec{s} = \theta}$	<p>The magnetic voltage <math>\theta</math> can be given as</p> <ul style="list-style-type: none"> <li>• <math>\theta = I</math> for a single conductor</li> <li>• <math>\theta = N \cdot I</math> for a coil</li> <li>• <math>\theta = \sum_n I_n</math> for multiple conductors</li> <li>• <math>\theta = \int_A \vec{S} \cdot d\vec{A}</math> for any spatial distribution (see <a href="#">block15</a>)</li> </ul>
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The unit of the magnetic voltage  $\theta$  is **Ampere** (or **Ampere-turns**).

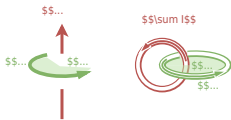
In the english literature the magnetic voltage is called **Magnetomotive force**

**Notice:**

$\oint \vec{s}$  and  $\int \vec{A}$  in  $\oint \vec{H} \cdot d\vec{s} = \theta = \int_A \vec{S} \cdot d\vec{A}$  build a right-hand system.

1. Once the thumb of the right hand is pointing along  $\int \vec{A}$ , the fingers of the right hand show the correct direction for  $\oint \vec{s}$  for positive  $\vec{H}$  and  $\vec{S}$
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



## Recap of the fieldline images

### longitudinal coil

Fig. 2: Magnetic field in a longitudinal coil

A longitudinal coil can be seen in [figure 2](#).

The created field density of the coil can be derived from Ampere's Circuital Law

$$\begin{aligned} \oint \vec{H} \cdot d\vec{s} &= \int_{\text{inner}} \vec{H} \cdot d\vec{s} + \int_{\text{outer}} \vec{H} \cdot d\vec{s} \\ &= \int \vec{H} \cdot d\vec{s} + 0 \\ &= \int \vec{H} \cdot d\vec{s} \end{aligned}$$

The magnetic field in a toroidal coil is often considered as homogenous in the inner volume, when the length  $l$  is much larger than the diameter:  $l \gg d$ .

With a given number  $N$  of windings, the magnetic field strength  $H$  is

### toroidal coil

Fig. 3: Magnetic field in a toroidal coil

A toroidal coil has a donut-like setup. This can be seen in [figure 3](#).

The toroidal coil is often defined by:

- The minor radius  $r$ : The radius of the circular cross-section of the coil.
- The major radius  $R$ : The distance from the center of the entire toroid (the center of the hole) to the center of the circular cross-section of the coil.

For reasons of symmetry, it shall get clear that the field lines form concentric circles. Also the magnetic field strength  $H$  in a toroidal coil is often considered as homogenous, when  $R \gg r$ . With a given number  $N$  of windings, the magnetic field strength  $H$  is

$$\begin{aligned} \theta &= H \cdot l = N \cdot I \\ H &= \frac{N \cdot I}{l} \end{aligned} \quad \bigg| \quad \text{longitudinal coil}$$

$$\begin{aligned} \theta &= H \cdot 2\pi R = N \cdot I \\ H &= \frac{N \cdot I}{2\pi R} \end{aligned} \quad \bigg| \quad \text{toroidal coil}$$

## Common pitfalls

- ...

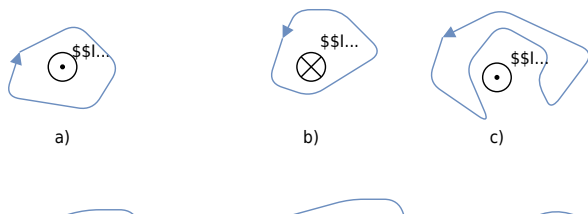
## Exercises

### Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors

Result e)

e)  $V_{(r, \theta)} = 20 \ln 2 = 13.86 \text{ V} \quad I_1 = 2 \text{ A} \quad I_2 = 4.5 \text{ A}$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let  $I_1 = 2 \text{ A}$  and  $I_2 = 4.5 \text{ A}$  be valid.

In each case, the magnetic potential difference  $V_{\text{m}}$  along the drawn path is sought.

Path

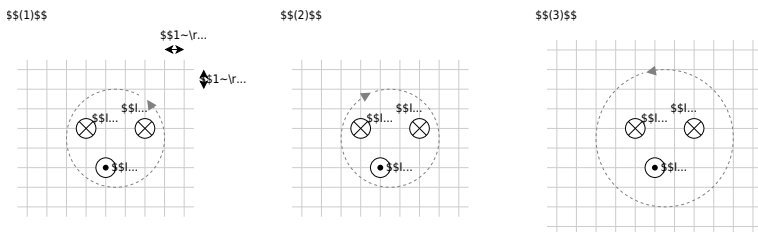
- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

**Exercise E1 Magnetic Voltage**  
**(written test, approx. 6 % of a 120-minute written test, SS2021)**

The following images show cross-sections of electrical cables.  
 A closed path is shown as a dashed line. The magnetic voltage  $\theta$  on these paths shall be analyzed.

The following values are given for the currents:

- $I_1 = 5 \text{ A}$
- $I_2 = 1 \text{ A}$
- $I_3 = 1 \text{ A}$
- $I_4 = 4 \text{ A}$



Specify which magnetic voltages  $\theta_{(1)}$ ,  $\theta_{(2)}$ , and  $\theta_{(3)}$  result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A} - 5 \text{ A} - 1 \text{ A}$
- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$

### Exercise E1 Magnetic Potential

**(written test, approx. 8 % of a 120-minute written test, SS2024)**

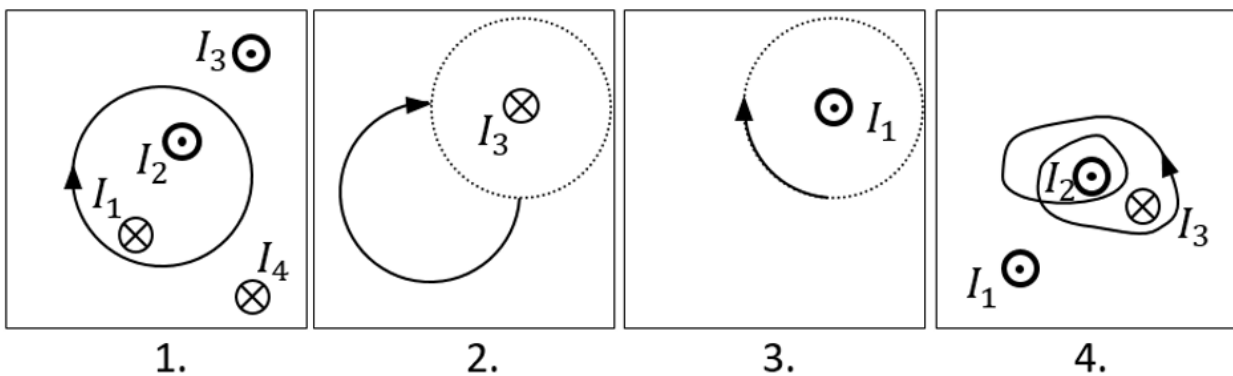
Calculate the magnetic potential difference  $V_{\text{m}}$  for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1| = 2 \text{ A}$
- $|I_2| = 5 \text{ A}$
- $|I_3| = 11 \text{ A}$
- $|I_4| = 7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task:  $+I_1 - I_2 = -3 \text{ A}$
2. Task:  $+\frac{1}{4} I_3 = 11/4 \text{ A}$  (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
3. Task:  $-\frac{1}{4} I_1 = -0.5 \text{ A}$
4. Task:  $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ A}$

**Exercise E6 Fields of an coax Cable**  
**(written test, approx. 12 % of a 120-minute written test, SS2024)**

2. With the graph of the magnitude of  $D$  ( $r$ ) with parameters  $D_0 = 0.6 \text{ V/m}$  and  $D_1 = 0.6 \text{ V/m}$  shown, the cross-section of the coax cable with  $(0, 0)$  as center of the cable is depicted in the diagram and labeled for the calculation appears:

Path

- Inner conductor:  $+3.3 \text{ mA}$ ,  $+10 \text{ nC}$  (current into the plane of the path diagram)
- Outer conductor:  $-3.3 \text{ mA}$ ,  $0 \text{ nC}$  (current out of the plane of diagram)
- for  $(0.1 \text{ mm} | 0)$ :  $B_{\text{I}} = 5.28 \dots \text{ A/m}^2$
- for  $(0.55 \text{ mm} | 0)$ :  $B_{\text{O}} = 6.985 \dots \text{ A/m}^2$

The magnitude of the electric displacement field  $D$  can be calculated by:  $\int D \cdot dA = Q_{\text{enc}}$ .

Here, in any position  $r$  from the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

For the shell as a region, the surface of the cylinder is  $A = 2\pi r \cdot l$ . This leads to:  $D(r) = \frac{Q_{\text{enc}}}{A} = \frac{Q_{\text{enc}}}{2\pi r \cdot l}$ .

This is proportional to the area within this radius. Therefore, the formula  $H = \frac{I_{\text{enc}}}{2\pi r}$  is used. So, we get for  $D_{\text{I}}(r)$  at  $r = 0.1 \text{ mm}$ , and  $D_{\text{O}}(r)$  at  $r = 0.55 \text{ mm}$ .

For  $r$  within the outer conductor one also gets a linear proportionality with a similar approach:  $D(r) = \frac{Q_{\text{enc}}}{2\pi r \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot (0.1 \cdot 10^{-3} \text{ m}) \cdot 0.5 \cdot 10^{-3} \text{ m}} \parallel D_{\text{O}}(r) = \frac{Q_{\text{enc}}}{2\pi r \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot (0.55 \cdot 10^{-3} \text{ m}) \cdot 0.5 \cdot 10^{-3} \text{ m}}$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the  $D$ -field is positive. But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength  $H$  at  $(0.1 \text{ mm} | 0)$  and  $(0.55 \text{ mm} | 0)$ ?

Path

The magnitude of the magnetic field strength  $H$  can be calculated by:  $H = \frac{I}{2\pi r}$

So, we get for  $H_{\text{I}}$  at  $(0.1 \text{ mm} | 0)$ , and  $H_{\text{O}}$  at  $(0.55 \text{ mm} | 0)$

~mm | 0)\$:

$$\begin{aligned} H_{\text{r m i}} &= \frac{I}{2 \pi \cdot r_{\text{r m i}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{ 0.1 \cdot 10^{-3} \text{ m} \}} \quad H_{\text{r m o}} &= \frac{I}{2 \pi \cdot r_{\text{r m o}}} \quad \&= \frac{+3.3 \text{ A}}{2 \pi \cdot \{ 0.55 \cdot 10^{-3} \text{ m} \}} \end{aligned}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the  $H$ -field on the right side points downwards.

Therefore, the sign of the  $H$ -field is negative.

But here, only the magnitude was questioned!

## Embedded resources

Explanation (video): ...

From:

<https://wiki.mexle.org/> - **MEXLE Wiki**

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[https://wiki.mexle.org/electrical\\_engineering\\_and\\_electronics\\_1/block16?rev=1765890556](https://wiki.mexle.org/electrical_engineering_and_electronics_1/block16?rev=1765890556)

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