Block 17 — Magnetic Flux Density and Forces

Student Group

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Learning objectives

After this 90-minute block, you can

- …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

We know from block11 that a static charge Q_1 generate a static electric field D.
Before in block09, we developed that a static electric field $E= {{1}\over{\varepsilon}} D$ effects a force F_C on a static charge Q_2

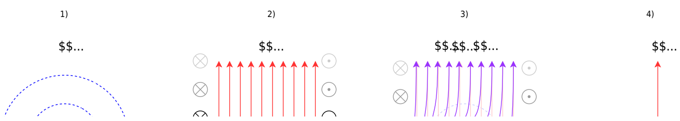
From the last chapter (block16) we got, that moving charges ${{d}\over{dt}}Q_1 = I_1$ generate a static magnetic field H.
So, how does an acting magnetic field effects a force on a moving charge ${{d}\over{dt}}Q_2 = I_2$?

Definition of the Magnetic Flux Density

To derive the forces, we do a step back to the images of field lines.
In figure 1 a) the field lines of a single current-carrying wire is shown.
figure 1 b) depicts the homogenous field of a coil.

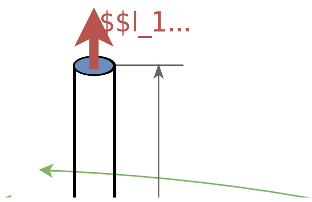
When a current-carrying wire is within the homogenous field, we get the superimposed picture of both fields.
This leads to an enrichment of magnetic field on the left and an depletion on the right.
With the knowledge, that the field lines usually do not like to stay next to each other on can conclude that there will be a force to the right.

Fig. 1: Force in magnetic field



To do so, the effect between two parallel conductors has to be examined closer.
The experiment consists of a part l of two very long[1] conductors with the different currents I_1, I_2 in the distance r (see figure 2).

Fig. 2: Forces between two Conductors



When no current is flowing through the conductors the forces er equal to zero.
Once the currents flow in the same direction (e.g. $I_1>0, I_2>0$) attracting forces $\vec{F}_{12} = - \vec{F}_{21}$ appear.
The force $\vec{F}_{xy}$ shall be the force on the conductor x caused by conductor y. In the following the force $\vec{F}_{12}$ on the conductor 1 will be examined.

The following is detectable:

1. $|\vec{F}_{12}| \sim I_1$, $|\vec{F}_{12}| \sim I_2$: The stronger each current, the stronger the force F_{12}.
2. $|\vec{F}_{12}| \sim l$: As longer the conductor length l, as stronger the force F_{12} gets.
3. $|\vec{F}_{12}| \sim {{1}\over{r}}$: A smaller distance r leads to stronger force F_{12}.

To summarize: \begin{align*} {F}_{12} \sim {I_1 \cdot I_2 \cdot {{l}\over{r}}} \end{align*}

The proportionality factor is arbitrarily chosen as: $$\begin{align*} {{{F}_{12} \cdot r} \over {I_1 \cdot I_2 \cdot {l}}} = {{\mu}\over{2\pi}} \end{align*}$$

Here μ is the magnetic permeability and for vacuum (vacuum permeability): $$\begin{align*} \mu = \mu_0 = 4\pi \cdot 10^{-7} {{\rm Vs}\over{\rm Am}} = 1.257 \cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \end{align*}$$

This leads to the **Ampere's Force Law**: $$\begin{align*} \boxed{ |\vec{F}_{12}| = {{\mu}\over{2 \pi}} \cdot {{I_1 \cdot I_2 }\over{r}} \cdot l } \end{align*}$$

Since we want to investigate the effect on the current I_1, the following rearrangement can be done: $$\begin{align*} |\vec{F}_{12}| &= {{\mu}\over{2 \pi}} \cdot {{I_2 }\over{r}} &\cdot I_1 \cdot l \\ &= B &\cdot I_1 \cdot l \\ \end{align*}$$

The properties of the field from I_2 acting on I_1 are summarized to B - the **magnetic flux density**.
B has the unit: $$\begin{align*} [B] &= {{[F]}\over{[I]\cdot[l]}} = 1 \rm {{N}\over{Am}} = 1 {{{VAs}\over{m}}\over{Am}} = 1 {{Vs}\over{m^2}} \\ &= 1 {\rm T} \quad\quad({\rm Tesla}) \end{align*}$$

This formula can be generalized with the knowledge of the directions of the conducting wire $\vec{l}$, the magnetic field strength $\vec{B}$ and the force $\vec{F}$ using vector multiplication too:

$$\begin{align*} \boxed{\vec{F_L} = I \cdot \vec{l} \times \vec{B}} \end{align*}$$

The absolute value can be calculated by

$$\begin{align*} \boxed{|\vec{F_L}| = I \cdot |l| \cdot |B| \cdot \sin(\angle \vec{l},\vec{B})} \end{align*}$$

The force is often called **Lorentz Force** F_L. For the orientation, another right-hand rule can be applied.

Notice:

Right-hand rule for the Lorentz Force:

- The causing current I is on the thumb. Since the current is not a vector, the direction is given by the direction of the conductor $\vec{l}$
- The mediating external magnetic field $\vec{B}$ is on the index finger
- The resulting force $\vec{F}$ on the conductor is on the middle finger

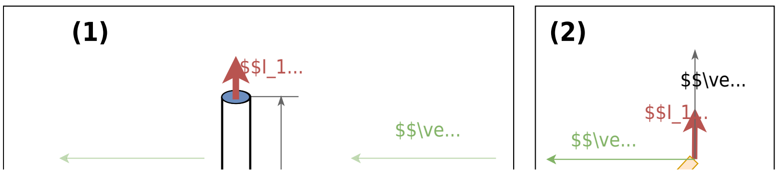
This is shown in figure 2.

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- $\vec{F}$orce on middle finger
- $\vec{B}$-Field on index finger
- Current I on thumb (direction with length $\vec{l}$)

To view the animation: click here!

Fig. 2: Force onto a single Conductor in a B-Field



Lorentz Law and Lorentz Force

The true Lorentz force is not the force on the whole conductor but the single force onto an (elementary) charge.
To find this force the previous force onto a conductor can be used as a start. However, the formula will be investigated infinitesimally for small parts ${\rm d} \vec{l}$ of the conductor:

$$\begin{align*} \vec{{\rm d}F}_{\rm L} = I \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}$$

The current is now substituted by $I = {\rm d}Q/{\rm d}t$, where ${\rm d}Q$ is the small charge packet in the length $\vec{{\rm d}l}$ of the conductor.

\begin{align*} \vec{{\rm d}F}_{\rm L} = {{{\rm d}Q}\over{{\rm d}t}} \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

\begin{align*} \vec{{\rm d}F}_{\rm L} &= {{{\rm d}Q \cdot {\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ \end{align*}

Here, the part ${{{\rm d}\vec{l}}\over{{\rm d}t}}$ represents the speed $\vec{v}$ of the small charge packet ${\rm d}Q$.

\begin{align*} \vec{{\rm d}F}_{\rm L} &= {\rm d}Q \cdot \vec{v} \times \vec{B} \end{align*}

The **Lorenz Force** on a finite charge packet is the integration:

\begin{align*} \boxed{\vec{F}_{\rm L} = Q \cdot \vec{v} \times \vec{B}} \end{align*}

Notice:

- A charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force of $\vec{F_{\rm L}}$.
- The direction of the force is given by the right-hand rule.

Please have a look at the German contents (text, videos, exercises) on the page of the KIT-Brückenkurs >> Lorentz-Kraft. The last part “Magnetic field within matter” can be skipped.

Common pitfalls

- …

Exercises

Exercise E7 Cylindrical Coil
(written test, approx. 6 % of a 120-minute written test, SS2021)

A cylindrical coil with the following information is given:
a) the magnetic flux density (2 points)
Result

- Length $□ = 30 {~\rm cm}$,
- Winding diameter $□ = 390 {~\rm mm}$,
- Number of windings $□ = 240$,
- Current through the conductor $□ = 500 {~\rm mA}$,
- Material inside: Air

Path

$\Phi = 4.60 ~\rm mWb$

- $\mu_0 = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

The magnetic flux density is $B = \mu_0 \mu_{\rm r} \cdot H$.

\begin{align*} B = \mu_0 \cdot H \end{align*}

\begin{align*} \Phi = B \cdot A \end{align*}

a) the magnetic field strength (2 points)

Since the coil is cylindrical, the cross-sectional area is given as:

Putting in the numbers: \begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 400 ~\rm {{A}\over{m}} \\ &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \end{align*}

Path

\begin{align*} A = \pi r^2 = \pi \left({{d}\over{2}} \right)^2 \end{align*}

Therefore: \begin{align*} \Phi &= B \cdot \pi \left({{d}\over{2}} \right)^2 \end{align*}

\begin{align*} H &= {{N \cdot I}\over{l}} \end{align*}

Putting in the numbers: \begin{align*} \Phi &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \cdot \pi \left({{0.39{\rm m}}\over{2}} \right)^2 \\ &= 0.00006004... {\rm Vs} \end{align*}

Putting in the numbers: \begin{align*} H &= {{240 \cdot 0.5 {~\rm A}}\over{0.3 {~\rm m}}} \end{align*}

Exercise E3 Magnetic Flux Density
(written test, approx. 6 % of a 120-minute written test, SS2021)

An electric motor is operated for experiments in the laboratory. An alternating current with an amplitude of $\hat{I} = 100~\rm A$ is operated.

You stand next to it and think about whether you have any health problems to worry about.

The figure below shows the top view of the laboratory with the supply line between $\rm A$ and $\rm B$.

a) The limit value for the magnetic flux density at the frequency used is $B_0 = 100~\rm µT$.

At what distance around the conductor is this value exceeded? (3 points, independent of a))

Result

Path

$B = 0.2 ~\rm mT$

$r = 42 ~\rm mm$

$\mu_{0} = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$, $\mu_{r}=1$

The formula for the magnetic field strength can be rearranged: \begin{align*} H &= {{I}\over{2\pi \cdot r}} \\ r &= {{I}\over{2\pi \cdot H}} \\ \end{align*}

Again, the magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Therefore: \begin{align*} r &= \mu_0 \mu_r {{ I }\over{2\pi \cdot B}} \\ &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} {{100 ~\rm A}\over{2\pi \cdot 100\cdot 10^{-6} {~\rm T}}} \\ \end{align*}

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$\begin{align*} H &= {{I}\over{2\pi \cdot r}} \end{align*}$$

The magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Here, the maximum current is $\hat{I} = 100~\rm A$ and the distance to the cable is $r = \sqrt{(0.1 {~\rm m})^2 + (0.4 {~\rm m})^2}= 0.412... ~\rm m$.

Therefore: $$\begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 1 \cdot {{100 ~\rm A}\over{2\pi \cdot 0.412... ~\rm m}} \end{align*}$$

Exercise E5 Toroidal Coil
(written test, approx. 5 % of a 120-minute written test, SS2021)

A magnetic field with a flux density of at least $50 ~\rm mT$ is to be achieved in a ring-shaped coil (toroidal coil).
The coil has 60 turns, wound around soft iron with $\mu_{\rm r} = 1200$.
The average field line length in the coil should be $l = 12 ~\rm cm$.

Result

$\mu_{0} = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

$I = 66 ~\rm mA$

What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

\begin{align*} H &= {{N \cdot I}\over{l}} \end{align*}

Based on the flux density the magnetic field strength can be derived by $B = \mu_0 \mu_{\rm r} \cdot H$.
By this, the formula can be rearranged:

\begin{align*} H &= {{N \cdot I}\over{l}} \\ {{B}\over{ \mu_0 \mu_{\rm r}}} &= {{N \cdot I}\over{l}} \\ I &= {{B \cdot l}\over{ \mu_0 \mu_{\rm r} \cdot N}} \end{align*}

Putting in the numbers: \begin{align*} I &= {{ 0.05 {~\rm T} \cdot 0.12{~\rm m} }\over{ 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 1'200 \cdot 60}} \\ &= 0.6631... {{\rm T\cdot m}\over{ {{\rm Vs}\over{\rm Am}} }} &= 0.6631... {{\rm {{\rm Vs}\over{\rm m^2}} \cdot m}\over{ {{\rm Vs}\over{\rm Am}} }} &= 0.6631... ~\rm A \end{align*}

Exercise E1 Lorentz Force (hard!)
(written test, approx. 10 % of a 120-minute written test, SS2021)

A $300 ~\rm km$ long, straight high-voltage direct current transmission line shall be analyzed. A current of $I = 1'200 ~\rm A$ flows through it.
A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_{\rm v} = 40 ~\rm \mu T$ and a horizontal component of $B_{\rm h} = 20 ~\rm \mu T$.
The angle between the transmission line and the horizontal component of the field strength is $\alpha = 20°$.
The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

1) The picture below shows the top view again. In which of the directions shown does the horizontal component of $F_{\rm h}$ of the resulting force act? (Independent)

Result

Only $\vec{F}_{\rm h}$ is perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$ and points in the right direction by the right-hand rule.

Front Vi

a) Calculate the force that results from the current flow on the entire conductor.
First, calculate the vertical and horizontal components and combine them accordingly.

Path

Top View

Path

The force on the transmission line can be calculated via the Lorentz force $\vec{F}_\rm L$: \begin{align*} \vec{F} = I \cdot (\vec{l} \times \vec{B}) \end{align*}

Here, we have two components for the current - and therefore for the force - to evaluate.
Considering the right-hand rule (and the cross product), the vertical field $B_{\rm v}$ generates a horizontal force $F_{\rm h}$ and vice versa.

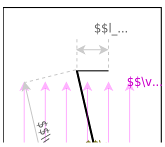
The **horizontal component** is given by

- The horizontal component $\vec{F}_{\rm h}$ of the force is based on the vertical component $\vec{B}_{\rm v}$ of the magnetic field.
- The vertical component $\vec{B}_{\rm v}$ of the magnetic field is not shown in the image but is pointing into the ground.
- It has to be perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$. The right-hand rule has to be applied.

\begin{align*} F_{\rm h} &= I \cdot (l \cdot B_{\rm v}) \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \\ &= 14'400 ~\rm {{VAs}\over{m}} = 14'400 ~\rm {{Ws}\over{m}} = 14'400 ~\rm N \end{align*}

For the **vertical component** the angle &\alpha& has to be considered.
For the maximum $F_{\rm v}$ the angle &\alpha& has to be $90°$, therefore the $\sin$ has to be used.

Top View



\begin{align*} F_{\rm v} &= I \cdot l \cdot B_{\rm h} \cdot \sin\alpha \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \cdot \sin 20° \\ &= 2'462.545... ~\rm N \end{align*}

For the **overall force** F the Pythagorean theorem has to be used:

\begin{align*} F &= \sqrt{F_{\rm v}^2 +F_{\rm h}^2} \\ &= \sqrt{({14'400 ~\rm N})^2 +({2'462.545... ~\rm N})^2} \\ &= 14'609.04... ~\rm N \end{align*}

Embedded resources

Explanation (video): …

[1] ideally: infinite long; in reality much longer, than the distance between them