Block 17 — Magnetic Flux Density and Forces

Student Group

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Learning objectives

After this 90-minute block, you can

- …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

We know from block11 that a static charge Q_1 generate a static electric field D.
Before in block09, we developed that a static electric field $E= {{1}\over{\varepsilon}} D$ effects a force F_C on a static charge Q_2

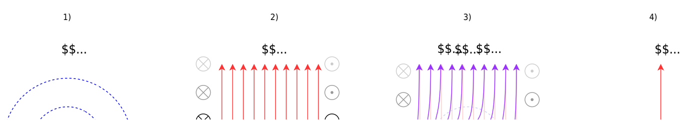
From the last chapter (block16) we got, that moving charges ${{d}\over{dt}}Q_1 = I_1$ generate a static magnetic field H.
So, how does an acting magnetic field effects a force on a moving charge ${{d}\over{dt}}Q_2 = I_2$?

Definition of the Magnetic Flux Density

To derive the forces, we do a step back to the images of field lines.
In figure 1 a) the field lines of a single current-carrying wire is shown.
figure 1 b) depicts the homogenous field of a coil.

When a current-carrying wire is within the homogenous field, we get the superimposed picture of both fields.
This leads to an enrichment of magnetic field on the left and an depletion on the right.
With the knowledge, that the field lines usually do not like to stay next to each other, one can conclude that there will be a force to the right.

Fig. 1: Force in magnetic field



When no current is flowing through the conductor the force is equal to zero.
The following is detectable:

1. $|\vec{F}| \sim I$: The stronger the current, the stronger the force F.
2. $|\vec{F}| \sim l$: As longer the conductor length l, as stronger the force F gets.
3. $|\vec{F}| \sim {H}$: As more current through the coil, as stronger the H-field. And a stronger the H-field leads to stronger force F.

To summarize: $$F \sim H \cdot I \cdot l$$

The proportionality factor is μ_0, the **magnetic field constant**, **permeability** or vacuum permeability: $\mu_0 = 4\pi \cdot 10^{-7} {\rm {Vs}\over{Am}}$.

$$F = \mu_0 \cdot H \cdot I \cdot l$$

When adding an iron core into the coil the force F gets stronger. Therefore, we include a material-dependent constant μ_r, the so-called relative permeability

$$\begin{align*} F = \mu_0 \mu_r \cdot H \cdot I \cdot l \\ \end{align*}$$

The new field quantity is B the **magnetic flux density**:

$$\begin{align*} \boxed{ \vec{B} = \mu \cdot \vec{H} } \quad | \text{with } \mu = \mu_0 \mu_r \end{align*}$$

Investigating the vectorial behaviour leads to the cross-product, and to the so-called **Lorentz force**

$$\begin{align*} \boxed{ \vec{F}_L = I \cdot \vec{l} \times \vec{B} } \end{align*}$$

With $\vec{l}$ pointing in the direction of the positive current I. The absolute value can be calculated by

$$\begin{align*} \boxed{|\vec{F_L}| = I \cdot |l| \cdot |B| \cdot \sin(\angle \vec{l},\vec{B})} \end{align*}$$

For the orientation of the vectors, another right-hand rule can be applied.

Notice:

Right-hand rule for the Lorentz Force:

- The causing current I is on the thumb. Since the current is not a vector, the direction is given by the direction of the conductor $\vec{l}$
- The mediating external magnetic field $\vec{B}$ is on the index finger
- The resulting force $\vec{F}$ on the conductor is on the middle finger

This is shown in figure 2.

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- $\vec{F}$orce on middle finger
- $\vec{B}$-Field on index finger
- Current I on thumb (direction with length $\vec{l}$)

To view the animation: click here!

Fig. 2: Force onto a single Conductor in a B-Field

Materials

The material can be divided into different types by looking at its relative permeability. figure 3 shows the relative permeability in the **magnetization curve** (also called B-H-curve). In this diagram, the different effect (B-field on y-axis) based on the causing external H-field (on x-axis) for different materials is shown. The three most important material types shall be discussed shortly.

Fig. 3: Magnetization Curve of different materials

Diamagnetic Materials

- Diamagnetic materials weaken the magnetic field, compared to the vacuum.
- The weakening is very low (see table 1).
- For diamagnetic materials applies $0<\mu_{\rm r}<1$
- The principle behind the effect is based on quantum mechanics (see figure 4):

Paramagnetic Materials

- Paramagnetic materials strengthen the magnetic field, compared to the vacuum.
- The strengthening is very low (see table 2).
- For paramagnetic materials applies $\mu_{\rm r}>1$
- The principle behind the effect is again based on quantum mechanics

Ferromagnetic Materials

- Ferromagnetic materials strengthen the magnetic field strongly, compared to the vacuum.
- The strengthening can create a field multiple times stronger than in a vacuum.
- For ferromagnetic materials applies $\mu_{\rm r} \gg 1$
- Ferromagnetic materials

    - Without the external field no counteracting field is generated by the matter.
    - With an external magnetic field an antiparallel-orientated magnet is induced.
    - The reaction weakens the external field. This is similar to the weakening of the electric field by the dipoles of materials.
- Due to the repulsion of the outer magnetic field the material tends to move out of a magnetic field.
  For very strong magnetic fields small objects can be levitated (see clip).

Material	**Symbol**	**$\mu_{\rm r}$**
Antimon	$\rm Sb$	$0.999 946$
Copper	$\rm Cu$	$0.999 990$
Mercury	$\rm Hg$	$0.999 975$
Silver	$\rm Ag$	$0.999 981$
Water	$\rm H_2O$	$0.999 946$
Bismut	$\rm Bi$	$0.999 830$

Tab. 1: Diamagnetic Materials

Fig. 4: Magnetic field in diamagnetic materials

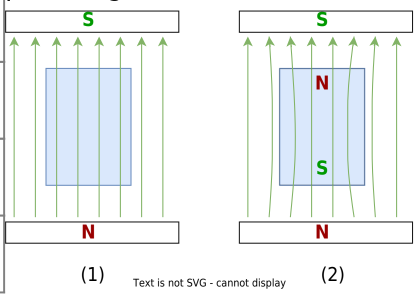
(see figure 5):

    - Without the external field no counteracting field is generated by the matter.
    - With an external magnetic field internal “tiny magnets” based on the electrons in their orbitals are orientated similarly.
    - This reaction strengthens the external field.

Material	**Symbol**	**$\mu_{\rm r}$**
Aluminum	$\rm Al$	$1.000 022$
Air		$1.000 000 4$
Oxygen	$\rm O_2$	$1.000 001 3$
Platinum	$\rm Pt$	$1.000 36$
Tin	$\rm Sn$	$1.000 003 8$

Tab. 2: Paramagnetic Materials

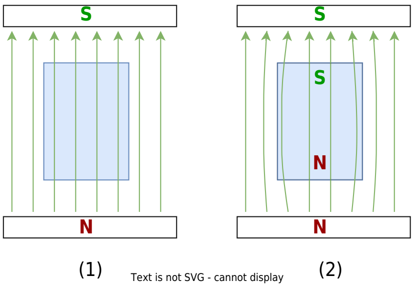
Fig. 5: Magnetic field in paramagnetic materials



are characterized by the magnetization curve (see figure 7)

    - Non-magnetized ferromagnets are located in the origin.
    - With an external field H the initial magnetization curve (in German: *Neukurve*, dashed in figure 7) is passed.
    - Even without an external field ($H=0$) and the internal field is stable.
      The stored field without external field is called **remanence** $B(H=0) = B_{\rm R}$ (or remanent magnetization).
    - In order to eliminate the stored field the counteracting **coercive field strength** $H_{\rm C}$ (also called coercivity) has to be applied.
    - The **saturation flux density** $B_{\rm sat}$ is the maximum possible magnetic flux density (at the maximum possible field strength $H_{\rm sat}$)

Fig. 7: Magnetization Curve



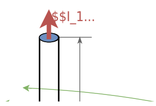
Applications of the Lorentz Force

We want to apply the Lorentz force for two common situations.

Two parallel Conductors

The Lorentz force can be applied to two parallel conductors.
The experiment consists of a part l of two very long[1)] and parallel conductors with the currents I_1, I_2 and the distance r (see figure 2).

Fig. 2: Forces between two Conductors



Here, we get for the B field caused by I_2: \begin{align*} B_2 &= \mu \cdot H_2 \\ &= \mu \cdot {{ I_2 }\over{2\pi \cdot r}} \end{align*}

We insert this into the formula of the Lorenz force \begin{align*} \vec{F}_L = I \cdot \vec{l} \times \vec{B} \end{align*}

This leads to the so-called **Ampere's Force Law**, applied on long and parallel conductors:

Moving single Charge

The true Lorentz force is not the force on the whole conductor but the single force onto an (elementary) charge.
To find this force the previous force onto a conductor can be used as a start. However, the formula will be investigated infinitesimally for small parts ${\rm d} \vec{l}$ of the conductor:

\begin{align*} \vec{{\rm d}F}_{\rm L} = I \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}

The current is now substituted by $I = {\rm d}Q/{\rm d}t$, where ${\rm d}Q$ is the small charge packet in the length $\vec{{\rm d}l}$ of the conductor.

\begin{align*} \vec{{\rm d}F}_{\rm L} = {{{\rm d}Q}\over{{\rm d}t}} \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

\begin{align*} \vec{{\rm d}F}_{\rm L} &= {{{\rm d}Q \cdot {\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ \end{align*}

Here, the part ${{{\rm d}\vec{l}}\over{{\rm d}t}}$ represents the speed $\vec{v}$ of the small charge packet ${\rm d}Q$.

\begin{align*} \vec{{\rm d}F}_{\rm L} &= {\rm

\begin{align*} \boxed{ |\vec{F}_{12}| = {{\mu}\over{2 \pi}} \cdot {{I_1 \cdot I_2 }\over{r}} \cdot l } \end{align*}

d}Q \cdot \vec{v} \times \vec{B} \end{align*}

The **Lorenz Force** on a finite charge packet is the integration:

\begin{align*} \boxed{\vec{F}_{\rm L} = Q \cdot \vec{v} \times \vec{B}} \end{align*}

Notice:

- A charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force of $\vec{F_{\rm L}}$.
- The direction of the force is given by the right-hand rule.

Please have a look at the German contents (text, videos, exercises) on the page of the KIT-Brückenkurs >> Lorentz-Kraft. The last part “Magnetic field within matter” can be skipped.

Common pitfalls

- …

Exercises

Exercise E1 Cylindrical Coil
(written test, approx. 6 % of a 120-minute written test, SS2021)

A cylindrical coil with the following information is given:

- Length $□ = 30 {~\rm cm}$,
- Winding diameter $□ = 390 {~\rm mm}$,
- Number of windings $□ = 240$,
- Current in the conductor $□ = 500 {~\rm mA}$,
- Material inside: Air
- $\mu_0 = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

a) the magnetic field strength (2 points)

b) the magnetic flux (2 points)

Result

Path

The magnetic flux is given as $B = \mu_0 \mu_{\rm r} \cdot H$:

\begin{align*} \Phi = B \cdot A \end{align*}

Since the coil is cylindrical, the cross-sectional area is given as

\begin{align*} A = \pi r^2 = \pi \left({{d}\over{2}} \right)^2 \end{align*}

\begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 400 ~\rm {{A}\over{m}} \\ &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \end{align*}

Path

Therefore: \begin{align*} \Phi &= B \cdot \pi \left({{d}\over{2}} \right)^2 \end{align*}
\begin{align*} H &= {{N \cdot I}\over{l}} = {{w \cdot I}\over{l}} \end{align*}
Putting in the numbers: \begin{align*} H &= {{240 \cdot 0.5 ~\rm A}}\over{0.3 ~\rm m}} \end{align*}
Putting in the numbers: \begin{align*} \Phi &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \cdot \pi \left({{0.39 {\rm m}}\over{2}} \right)^2 \\ &= 0.00006004... {\rm Vs} \end{align*}

Exercise E2 Magnetic Flux Density
(written test, approx. 6 % of a 120-minute written test, SS2021)

An electric motor is operated for experiments in the laboratory. An alternating current with an amplitude of $\hat{I} = 100~\rm A$ is operated.
You stand next to it and think about whether you have any health problems to worry about.
The figure below shows the top view of the laboratory with the supply line between $\rm A$ and $\rm B$.
$\mu_{0} = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$, $\mu_{r}=1$

a) The limit value for the magnetic flux density at the frequency used is $B_0 = 100~\rm \mu T$.
At what distance around the conductor is this value exceeded? (3 points, independent of a))

Result

$B = 42 ~\rm \mu T$

$r = 0.2 ~\rm m$

Path

The formula for the magnetic field strength can be rearranged: \begin{align*} H &= {{I}\over{2\pi \cdot r}} \\ r &= {{I}\over{2\pi \cdot H}} \\ \end{align*}

Again, the magnetic flux density B is given as: $B = \mu_0 \mu_r H$
Therefore: \begin{align*} r &= \mu_0 \mu_r {{ I }\over{2\pi \cdot B}} \\ &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} {{100 ~\rm A}\over{2\pi \cdot 100\cdot 10^{-6}}{~\rm T}}} \\ \end{align*}

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$\begin{align*} H &= {{I}\over{2\pi \cdot r}} \end{align*}$$

The magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Here, the maximum current is $\hat{I} = 100~\rm A$ and the distance to the cable is $r = \sqrt{(0.1 {~\rm m})^2 + (0.4 {~\rm m})^2}= 0.412... ~\rm m$.

Therefore: $$\begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 1 \cdot {{100 ~\rm A}\over{2\pi \cdot 0.412... ~\rm m}} \end{align*}$$

Exercise E1 Toroidal Coil
(written test, approx. 5 % of a 120-minute written test, SS2021)

A magnetic field with a flux density of at least $50 ~\rm mT$ is to be achieved in a ring-shaped coil (toroidal coil).
The coil has 60 turns, wound around soft iron with $\mu_{\rm r} = 1200$.
The average field line length in the coil should be $l = 12 ~\rm cm$.

$\mu_{0} = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

Result

$I = 66 ~\rm mA$

What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

\begin{align*} H &= {{N \cdot I}\over{l}} \end{align*}

Based on the flux density the magnetic field strength can be derived by $B = \mu_0 \mu_{\rm r} \cdot H$.
By this, the formula can be rearranged:

\begin{align*} H &= {{N \cdot I}\over{l}} \\ {{B}\over{ \mu_0 \mu_{\rm r}}} &= {{N \cdot I}\over{l}} \\ I &= {{B \cdot l}\over{ \mu_0 \mu_{\rm r} \cdot N}} \end{align*}

Putting in the numbers: \begin{align*} I &= {{ 0.05 {~\rm T} \cdot 0.12{~\rm m} }\over{ 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 1'200 \cdot 60}} \\ &= 0.6631... {{\rm T\cdot m}\over{ {{\rm Vs}\over{\rm Am}} }} &= 0.6631... {{\rm {{\rm Vs}\over{\rm m^2}} \cdot m}\over{ {{\rm Vs}\over{\rm Am}} }} &= 0.6631... ~\rm A \end{align*}

Exercise E1 Lorentz Force (hard!)
(written test, approx. 10 % of a 120-minute written test, SS2021)

A $300 ~\rm km$ long, straight high-voltage direct current transmission line shall be analyzed. A current of $I = {1'200} ~\rm A$ flows through it.
A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_{\rm v} = 40 ~\rm \mu T$ and a horizontal component of $B_{\rm h} = 20 ~\rm \mu T$.
The angle between the transmission line and the horizontal component of the field strength is $\alpha = 20 °$.
The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

b) The picture below shows the top view again. In which of the directions shown does the horizontal component of $F_{\rm h}$ of the resulting force act? (Independent)

Result

$F = 14'600 ~\rm mN$

Only ... is perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$ and points in the right direction by the right-hand rule.

a) Calculate the force that results from the current flow on the entire conductor.
First, calculate the vertical and horizontal components and combine them accordingly.

Path

Top View

Path

The force on the transmission line can be calculated via the Lorentz force $\vec{F}_\rm L$:

$$\begin{align*} \vec{F} = I \cdot (\vec{l} \times \vec{B}) \end{align*}$$

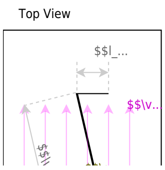
Here, we have two components for the current - and therefore for the force - to evaluate.
Considering the right-hand rule (and the cross product), the vertical field $B_{\rm v}$ generates a horizontal force $F_{\rm h}$ and vice versa.

- The horizontal component $\vec{F}_{\rm h}$ of the force is based on the vertical component $\vec{B}_{\rm v}$ of the magnetic field.
- The vertical component $\vec{B}_{\rm v}$ of the magnetic field is not shown in the image but is pointing into the ground.
- It has to be perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$. The right-hand rule has to be applied.

The **horizontal component** is given by

\begin{align*} F_{\rm h} &= I \cdot (l \cdot B_{\rm v}) \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \\ &= 14'400 ~\rm {{VAs}\over{m}} = 14'400 ~\rm {{Ws}\over{m}} = 14'400 ~\rm N \end{align*}

For the **vertical component** the angle &\alpha& has to be considered.
For the maximum $F_{\rm v}$ the angle &\alpha& has to be $90°$, therefore the $\sin$ has to be used.



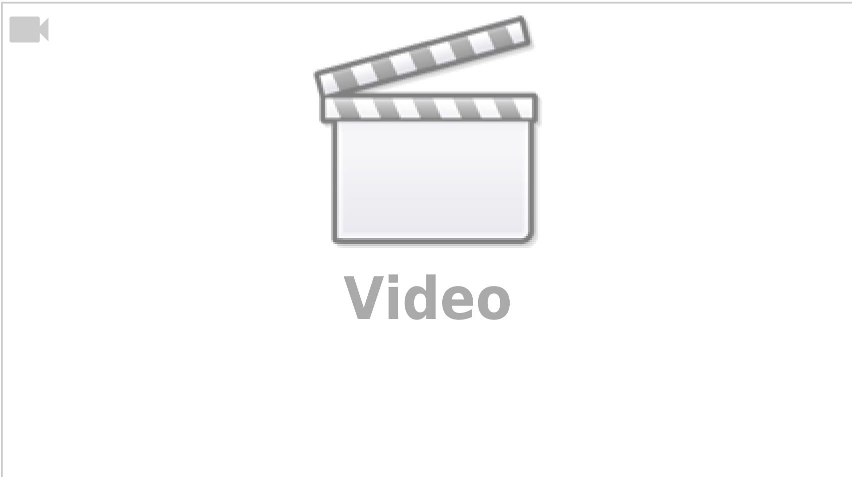
\begin{align*} F_{\rm v} &= I \cdot l \cdot B_{\rm h} \cdot \sin\alpha \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \cdot \sin 20° \\ &= 2'462.545... ~\rm N \end{align*}

For the **overall force** F the Pythagorean theorem has to be used:

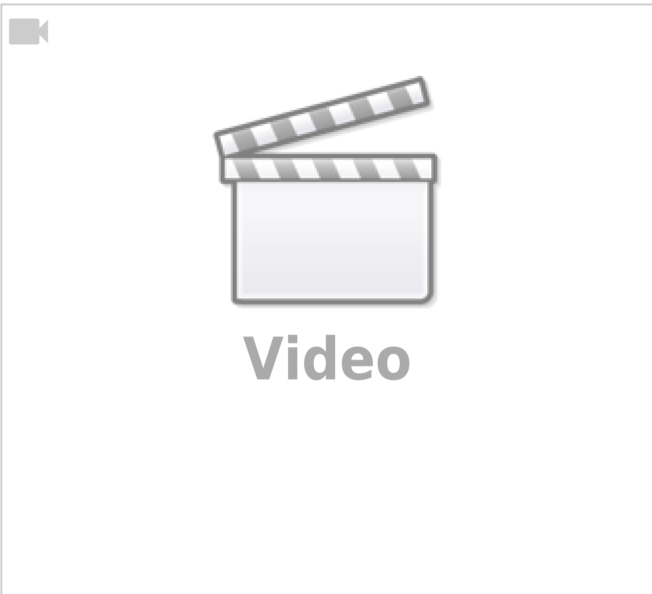
\begin{align*} F &= \sqrt{F_{\rm v}^2 +F_{\rm h}^2} \\ &= \sqrt{({14'400 ~\rm N})^2 +({2'462.545... ~\rm N})^2} \\ &= 14'609.04... ~\rm N \end{align*}

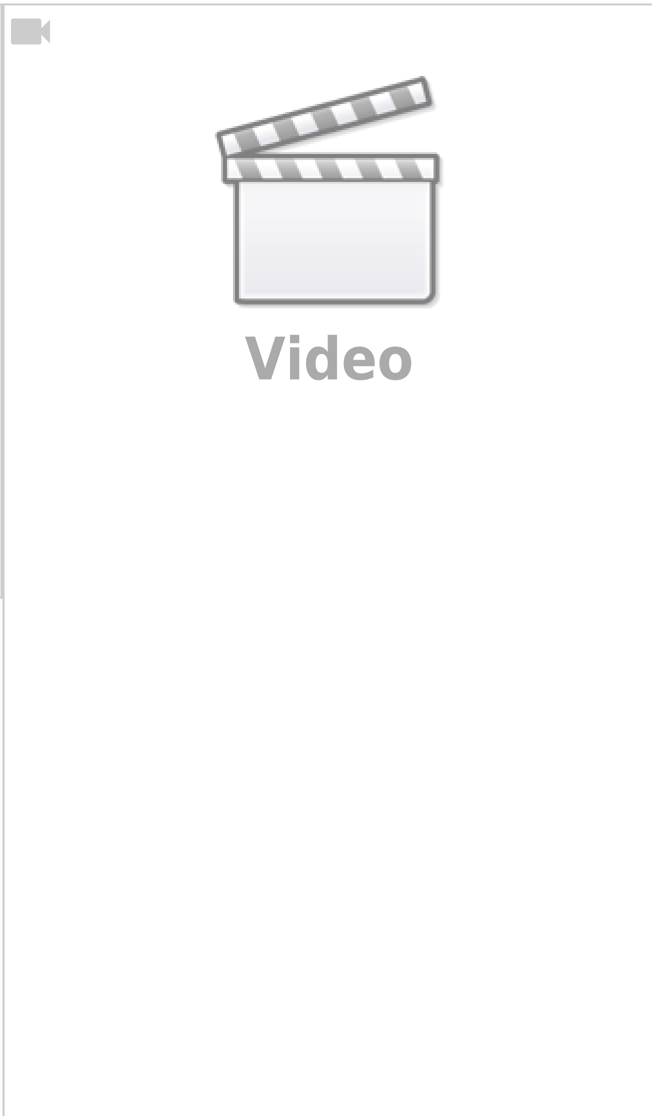
Embedded resources

A living insect ("diamagnet") floats in a very strong magnetic field



Explanation of diamagnetism and paramagnetism





1)
ideally: infinite long; in reality much longer, than the distance between them