

Block 17 — Magnetic Flux Density and Forces

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Block 17 — Magnetic Flux Density and Forces	3
Learning objectives	3
Preparation at Home	3
90-minute plan	3
Conceptual overview	3
Core content	3
Definition of the Magnetic Flux Density	4
Notice:	5
Materials	6
Diamagnetic Materials	7
Paramagnetic Materials	7
Ferromagnetic Materials	7
Applications of the Lorentz Force	9
Two parallel Conductors	9
Moving single Charge	9
Notice:	10
Common pitfalls	10
Exercises	10
Exercise E1 Cylindrical Coil (written test, approx. 6 % of a 120-minute written test, SS2021)	10
Exercise E2 Magnetic Flux Density (written test, approx. 6 % of a 120-minute written test, SS2021)	11
Exercise E1 Toroidal Coil (written test, approx. 5 % of a 120-minute written test, SS2021)	13
Exercise E1 Lorentz Force (hard!) (written test, approx. 10 % of a 120-minute written test, SS2021)	14
Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor	18
Task 3.3.1 magnetic Flux Density	18
Task 3.3.2 Electron in Plate Capacitor with magnetic Field	18

Embedded resources 22

Block 17 — Magnetic Flux Density and Forces

Learning objectives

After this 90-minute block, you can

- ...

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. ...

Core content

We know from [block11](#) that a static charge Q_1 generate a static electric field D .
Before in [block09](#), we developed that a static electric field $E = \frac{1}{\epsilon_0} D$ effects a force F_C on a static charge Q_2

From the last chapter ([block16](#)) we got, that moving charges $\frac{dQ_1}{dt} = I_1$ generate a static magnetic field H .

So, how does an acting magnetic field effects a force on a moving charge $\frac{dQ_2}{dt} = I_2$?

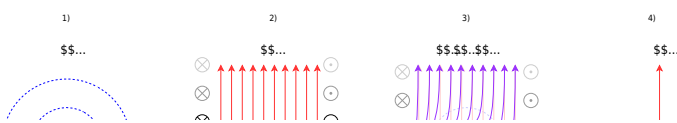
Definition of the Magnetic Flux Density

To derive the forces, we do a step back to the images of field lines.
 In [figure 1 1](#)) the field lines of a single current-carrying wire is shown.
[figure 1 2](#)) depicts the homogenous field of a coil.

When a current-carrying wire is within the homogenous field, we get the superimposed picture of both fields ([figure 1 3](#)).

This leads to an enrichment of magnetic field on the left and an depletion on the right.
 With the knowledge, that the field lines usually do not like to stay next to each other, one can conclude that there will be a force to the right.

Fig. 1: Force in magnetic field



When no current is flowing through the conductor the force is equal to zero.
 The following is detectable:

1. $|\vec{F}| \sim I$: The stronger the current, the stronger the force F .
2. $|\vec{F}| \sim l$: As longer the conductor length l , as stronger the force F gets.
3. $|\vec{F}| \sim H$: As more current through the coil, as stronger the H -field. And a stronger the H -field leads to stronger force F .

To summarize:
$$F \sim H \cdot l \cdot I$$

The proportionality factor is μ_0 , the **magnetic field constant, permeability** or **vacuum permeability**: $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$.

$$F = \mu_0 \cdot H \cdot l \cdot I$$

When adding an iron core into the coil the force F gets stronger. Therefore, we include a material-dependent constant μ_r , the so-called relative permeability

$$F = \mu_0 \mu_r \cdot H \cdot I \cdot l$$

The new field quantity is B the **magnetic flux density**:

$$\boxed{\vec{B} = \mu \cdot \vec{H}} \quad | \quad \text{with } \mu = \mu_0 \mu_r$$

Investigating the vectorial behaviour leads to the cross-product, and to the so-called **Lorentz force**

$$\boxed{\vec{F}_L = I \cdot \vec{l} \times \vec{B}}$$

With \vec{l} pointing in the direction of the positive current I . The absolute value can be calculated by

$$|\vec{F}_L| = I \cdot l \cdot B \cdot \sin(\angle \vec{l}, \vec{B})$$

For the orientation of the vectors, another right-hand rule can be applied.

Notice:

Right-hand rule for the Lorentz Force:

- The causing current I is on the thumb. Since the current is not a vector, the direction is given by the direction of the conductor \vec{l}
- The mediating external magnetic field \vec{B} is on the index finger
- The resulting force \vec{F} on the conductor is on the middle finger

This is shown in [figure 2](#).

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- \vec{F} force on middle finger
- \vec{B} -Field on index finger
- Current I on thumb (direction with length \vec{l})

To view the animation: [click here!](#)

Fig. 2: Force onto a single Conductor in a B-Field



Materials

The material can be divided into different types by looking at its relative permeability. [figure 3](#) shows the relative permeability in the **magnetization curve** (also called B - H -curve). In this diagram, the different effect (B -field on y -axis) based on the causing external H -field (on x -axis) for different materials is shown. The three most important material types shall be discussed shortly.

Fig. 3: Magnetization Curve of different materials

Diamagnetic Materials

- Diamagnetic materials weaken the magnetic field, compared to the vacuum.
- The weakening is very low (see [table 1](#)).
- For diamagnetic materials applies $0 < \mu_{\text{r}} < 1$
- The principle behind the

Paramagnetic Materials

- Paramagnetic materials strengthen the magnetic field, compared to the vacuum.
- The strengthening is very low (see [table 2](#)).
- For paramagnetic materials applies $\mu_{\text{r}} > 1$

Ferromagnetic Materials

- Ferromagnetic materials strengthen the magnetic field strongly, compared to the vacuum.
- The strengthening can create a field multiple times stronger than in a vacuum (see [figure 6](#)).
- For ferromagnetic

effect is based on quantum mechanics (see figure 4):

- Without the external field no counteracting field is generated by the matter.
- With an external magnetic field an antiparallel-orientated magnet is induced.
- The reaction weakens the external field. This is similar to the weakening of the electric field by the dipoles of materials.

- Due to the repulsion of the outer magnetic field the material tends to move out of a magnetic field. For very strong magnetic fields small objects can be levitated (see clip).

Material	Symbol	μ_{r}
Antimon	Sb	0.999 946
Copper	Cu	0.999 990
Mercury	Hg	0.999 975
Silver	Ag	0.999 981
Water	H_2O	0.999 946
Bismut	Bi	0.999 830

Tab. 1: Diamagnetic Materials

Fig. 4: Magnetic field in diamagnetic materials

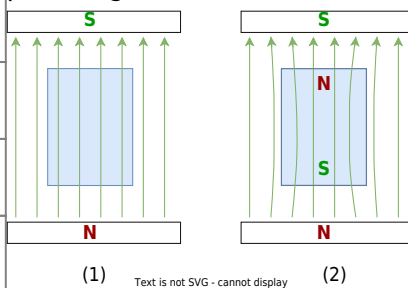
- The principle behind the effect is again based on quantum mechanics (see figure 5):

- Without the external field no counteracting field is generated by the matter.
- With an external magnetic field internal “tiny magnets” based on the electrons in their orbitals are orientated similarly.
- This reaction strengthens the external field.

Material	Symbol	μ_{r}
Aluminum	Al	1.000 022
Air		1.000 000 4
Oxygen	O_2	1.000 001 3
Platinum	Pt	1.000 36
Tin	Sn	1.000 003 8

Tab. 2: Paramagnetic Materials

Fig. 5: Magnetic field in paramagnetic materials



materials applies $\mu_{\text{r}} \gg 1$

- Ferromagnetic materials are characterized by the magnetization curve (see figure 7)

- Non-magnetized ferromagnets are located in the origin.
- With an external field H the initial magnetization curve (in German: *Neukurve*, dashed in figure 7) is passed.

- Even without an external field ($H=0$) and the internal field is stable. The stored field without external field is called **remanence** $B(H=0) = B_{\text{r}}$ (or remanent magnetization).

- In order to eliminate the stored field the counteracting **coercive field strength** H_{C} (also called coercivity) has to be applied.

- The **saturation flux density** B_{sat} is the maximum possible magnetic flux density (at the maximum possible field strength H_{sat})

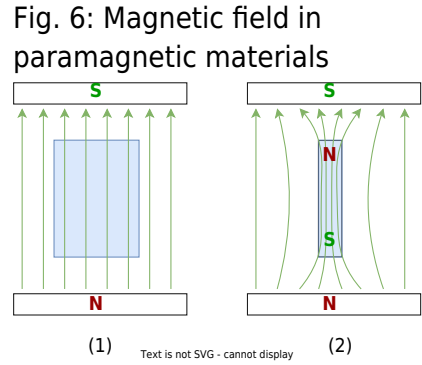


Fig. 7: Magnetization Curve

Applications of the Lorentz Force

We want to apply the Lorentz force for two common situations.

Two parallel Conductors

The Lorentz force can be applied to two parallel conductors. The experiment consists of a part of two very long¹⁾ and parallel conductors with the currents I_1, I_2 and the distance r (see figure 2).

Fig. 2: Forces between two Conductors

Moving single Charge

The true Lorentz force is not the force on the whole conductor but the single force onto an elementary charge. To find this force the previous force onto a conductor can be used as a start. However, the formula will be investigated infinitesimally for small parts $d\vec{l}$ of the conductor:

$$\vec{dF}_L = I \cdot d\vec{l} \times \vec{B}$$

The current is now substituted by $I = \frac{dQ}{dt}$, where dQ is the small charge packet in the length $d\vec{l}$ of the conductor.

$$\vec{dF}_L = \left\{ \frac{dQ}{dt} \cdot d\vec{l} \right\} \times \vec{B}$$

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

$$\vec{dF}_L = \left\{ \frac{dQ}{dt} \cdot d\vec{l} \right\} \times \vec{B} = dQ \cdot \left\{ \frac{d\vec{l}}{dt} \right\} \times \vec{B}$$



Here, we get for the B field caused by I_2 :
$$\vec{B} = \mu \cdot H_2 = \mu \cdot \frac{I_2}{2\pi r} \cdot \vec{l}$$

We insert this into the formula of the Lorentz force
$$\vec{F}_L = I \cdot \vec{l} \times \vec{B}$$

This leads to the so-called **Ampere's Force Law**, applied on long and parallel conductors:
$$|\vec{F}_{12}| = \frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2}{r} \cdot l$$

Here, the part $\frac{dl}{dt}$ represents the speed v of the small charge packet dQ .

$$\vec{F}_{\text{L}} = dQ \cdot \vec{v} \times \vec{B}$$

The **Lorentz Force** on a finite charge packet is the integration:

$$\vec{F}_{\text{L}} = Q \cdot \vec{v} \times \vec{B}$$

Notice:

- A charge dQ moving with a velocity \vec{v} in a magnetic field \vec{B} experiences a force of \vec{F}_{L} .
- The direction of the force is given by the right-hand rule.

Common pitfalls

- ...

Exercises

Exercise E1 Cylindrical Coil (written test, approx. 6 % of a 120-minute written test, SS2021)

A) The magnetic flux (2 points) information is given:

Result

- Length $l = 30 \text{ cm}$,

Path Winding diameter $d = 390 \text{ mm}$,

- Number of windings $N = 240$,
- Current in the conductor $I = 500 \text{ mA}$,

Material inside: Air

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$$

The magnetic field strength is $B = \mu_0 \cdot \mu_r \cdot H$:

The magnetic field due to the wire can be neglected. Determine the following for the inside of the coil:

But the coil is cylindrical, so $B = \mu_0 n I$ and $n = \frac{N}{l}$

$$B = \mu_0 \frac{N}{l} I = 4\pi \times 10^{-7} \frac{Vs}{Am} \cdot \frac{400}{0.3} \cdot I$$

$$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2$$

Therefore:

$$\Phi = B \cdot A = \mu_0 \frac{N}{l} I \cdot \pi \left(\frac{d}{2} \right)^2$$

Putting in the numbers:

$$\Phi = 0.0005026... \frac{Vs}{m^2} \cdot \pi \left(\frac{0.39 \text{ m}}{2} \right)^2 = 0.00006004... \text{ Vs}$$

$$B = \frac{\Phi}{A} = \frac{N \cdot I}{l} = \frac{w \cdot I}{l}$$

Putting in the numbers:

$$H = \frac{240 \cdot 0.5 \text{ A}}{0.3 \text{ m}}$$

Exercise E2 Magnetic Flux Density (written test, approx. 6 % of a 120-minute written test, SS2021)

A) The electric field is created for the experiment in the laboratory. A set of $B = 100 \text{ mT}$ with a magnitude of $I = 100 \text{ A}$ is operated.

Now stand next to it and think about what this value means. (B points in the \hat{z} direction). The figure below shows the top view of the laboratory with the supply line between A and B .

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}, \mu_r = 1$$

The formula for the magnetic field strength can be rearranged:

$$H = \frac{I}{2\pi r} \implies r = \frac{I}{2\pi H}$$

Again, the magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Therefore:

$$r = \frac{\mu_0 \mu_r I}{2\pi B} = \frac{4\pi \cdot 10^{-7} \frac{Vs}{Am} \cdot 100 \text{ A}}{2\pi \cdot 100 \cdot 10^{-6} \text{ T}} = 2 \cdot 10^{-7} \text{ m}$$

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$\begin{aligned} H &= \frac{I}{2\pi \cdot r} \end{aligned}$$

The magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Here, the maximum current is $\hat{I} = 100 \text{ ~\rm A}$ and the distance to the cable is $r = \sqrt{(0.1 \text{ ~\rm m})^2 + (0.4 \text{ ~\rm m})^2} = 0.412... \text{ ~\rm m}$.

$$\begin{aligned} B &= 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1 \\ &\cdot \frac{100 \text{ ~\rm A}}{2\pi \cdot 0.412... \text{ ~\rm m}} \end{aligned}$$

Exercise E1 Toroidal Coil**(written test, approx. 5 % of a 120-minute written test, SS2021)**

A magnetic field with a flux density of at least 50 mT is to be achieved in a ring-shaped coil (toroidal coil).

The coil has 60 turns, wound around soft iron with $\mu_r = 1200$.

The average field line length in the coil should be $l = 12 \text{ cm}$.

Result
 The minimum current is $I_{\min} = 4 \pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$



What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

$$\begin{aligned} H &= \frac{N \cdot I}{l} \end{aligned}$$

Based on the flux density the magnetic field strength can be derived by $B = \mu_0 \mu_r H$.

By this, the formula can be rearranged:

$$\begin{aligned} H &= \frac{N \cdot I}{l} \quad \parallel \quad \frac{B}{\mu_0 \mu_r} &= \\ \frac{N \cdot I}{l} &\parallel I &= \frac{B \cdot l}{\mu_0 \mu_r \cdot N} \\ \end{aligned}$$

Putting in the numbers:

$$\begin{aligned} I &= \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1'200 \cdot 60} \parallel &= \\ &= 0.6631... \frac{\text{T} \cdot \text{m}}{\frac{\text{Vs}}{\text{Am}}} &= 0.6631... \frac{\text{Vs}}{\text{m}^2} \cdot \text{m} \parallel &= \\ &= 0.6631... \text{ A} \end{aligned}$$

Exercise E1 Lorentz Force (hard!)

(written test, approx. 10 % of a 120-minute written test, SS2021)

A) ~~300 picture below shows straight high voltage direct wire of the dimensions shown in the picture. A current of $I = 200 \text{ A}$ flows through the wire.~~
 Result: A component of $F = 1.200 \text{ N}$ of the Lorentz force acts? (Independent)

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_v = 40 \mu\text{T}$ and a horizontal component of $B_h = 20 \mu\text{T}$.

Only a ~~1500~~ 7 cm long wire is perpendicular to \vec{B}_v and to \vec{B}_h and points in the right direction by the right-hand rule.

The angle between the transmission line and the horizontal component of the field strength is $\alpha = 20^\circ$.
 The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

a) Calculate the force that results from the current flow on the entire conductor.
 First, calculate the vertical and horizontal components and combine them accordingly.

Path
 Top View

Path

The force on the transmission line can be calculated via the Lorentz force

$$\vec{F} = I \cdot (\vec{l} \times \vec{B})$$

- The horizontal component F_h of the force is based on the vertical component B_v of the magnetic field.
- The vertical component F_v of the force is based on the horizontal component B_h of the magnetic field.

Here, we have two components for the current and therefore for the force - to evaluate.

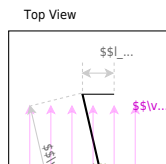
Considering the right-hand rule (and the cross product), the vertical field B_v generates a horizontal force F_h and vice versa.

The **horizontal component** is given by

$$\begin{aligned} F_{\text{h}} &= I \cdot (I \cdot B_{\text{v}}) \\ &= 1'200 \text{ A} \cdot 300 \\ &\cdot 10^3 \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \\ &= 14'400 \frac{\text{VA}}{\text{m}} = 14'400 \frac{\text{Ws}}{\text{m}} = 14'400 \text{ N} \end{aligned}$$

For the **vertical component** the angle α has to be considered.

For the maximum F_{v} the angle α has to be 90° , therefore the \sin has to be used.



$$\begin{aligned} F_{\text{v}} &= I \cdot I \cdot B_{\text{h}} \cdot \sin \alpha \\ &= 1'200 \text{ A} \cdot 300 \cdot 10^3 \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \\ &\cdot \sin 20^\circ = 2'462.545... \text{ N} \end{aligned}$$

For the **overall force** F the Pythagorean theorem has to be used:

$$\begin{aligned} F &= \sqrt{F_{\text{v}}^2 + F_{\text{h}}^2} \\ &= \sqrt{(14'400 \text{ N})^2 + (2'462.545... \text{ N})^2} \\ &= 14'609.04... \text{ N} \end{aligned}$$

Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor

The current is $I = 100\text{ A}$. Find the magnetic field strength H at a point P , which is outside the conductor. The conductor has a constant section with the radius $r = 1\text{ cm}$. The radius of the conductor is $r = 1\text{ cm}$.

What is the magnetic field strength H at a point P , which is outside the conductor?

A small loop of wire carrying a current I is placed near the conductor. The magnetic field strength H at a point P is given by the formula:

$$H = \frac{I}{2\pi r}$$

For comparison, the same flux density shall be created inside a toroidal coil with $N = 1000$ windings. The current I is given as $I = 10\text{ A}$. The cross-sectional area is $A = 5\text{ cm}^2$. The magnetic field is given as:

$$B = \mu_0 \cdot H \cdot N \cdot I$$

So the current I is given as:

$$I = \frac{B \cdot A}{\mu_0 \cdot N}$$

The H field is given as the current I through an area divided by the "specific" length l of the closed path around the area. This shall give you the formula (when not already known):

$$H = \frac{I}{l}$$

This can be rearranged to the current I as follows:

$$I = H \cdot l$$

The B field can be calculated by the H field:

$$B = \mu_0 \cdot H$$

- The H field is given as the current I through an area divided by the "specific" length l of the closed path around the area. This shall give you the formula (when not already known)
- The current is number of windings times I .

Task 3.3.2 Electron in Plate Capacitor with magnetic Field

An electron enters a plate capacitor on a trajectory parallel to the plates. It shall move with the velocity v in the plate capacitor parallel to the plates. The plates have a potential difference U and a distance d . In the vacuum in between the plates, there is also a magnetic field B present.

Fig. 9: Electron in B- and E-Field



Calculate the velocity depending on the other parameters $\vec{v} = f(U, |\vec{B}|, d)$!

Path

- Think about the two forces on the electron from the fields - gravity is ignored. Write their definitions down.
- With which relationship between these two forces does the electron moves through the plate capacitor parallel to the plates?
So the trajectory neither get bent up nor down.
- What is the relationship between the E -field in the plate capacitor and the electric voltage U ?

Solution

Within the electric field, the Coulomb force acts on the electron:

$$\vec{F}_C = q_e \cdot \vec{E}$$

Within the magnetic field, also the Lorentz force acts on the electron:

$$\vec{F}_L = q_e \cdot \vec{v} \times \vec{B}$$

The absolute value of both forces must be equal to compensate each other:

$$|\vec{F}_C| = |\vec{F}_L| \quad |q_e \cdot \vec{E}| = |q_e \cdot \vec{v} \times \vec{B}| \\ |q_e \cdot \vec{E}| = q_e \cdot |\vec{E}| \quad |q_e \cdot \vec{v} \times \vec{B}| = q_e \cdot |\vec{v} \times \vec{B}| \\ |\vec{E}| = |\vec{v} \times \vec{B}|$$

Since \vec{v} is perpendicular to \vec{B} the right side is equal to $|\vec{v}| \cdot |\vec{B}| = v \cdot B$.

Additionally, for the plate capacitor $|\vec{E}| = U/d$.

Therefore, it leads to the following:

$$\frac{U}{d} = v \cdot B \quad v = \frac{U}{B \cdot d}$$

Result

$$v = \frac{U}{B \cdot d}$$

Task 1

1. Which hand can be used to infer magnetic field direction from currents?

[Tip for 1](#)

- The right hand
- The left hand

For the current, you use which hand?

2. In the derivation from 1. how are the fingers to be assigned?

[Tip for 2](#)

- Thumb for current direction, remaining fingers for magnetic field direction
- Thumb for magnetic field direction, remaining fingers for current direction
- both possibilities are correct

- Imagine a coil with a winding pictorially, or draw it on.
- Now think of a generated field through this to it. What direction must the current flow, that causes the field? Does this fit the rule of thumb?
- Then try it the other way round: If a current is given, where do the field lines go in and where out? What poles are created there?

3. Two conductors carrying current are parallel and close to each other. The current in both is flowing in the same direction. What force effect can be seen?

- none
- The conductors attract

[Tip for 3](#)

- The conductors repel

See 3rd video.

4. Two conductors carrying current are at right angles to each other. Current flows through both of them. What force effect can be seen?

- none
- The conductors attract
- The conductors repel

5. What is the magnetic field inside the earth or a permanent magnet?

- from the magnetic north pole to the south pole
- from the magnetic south pole to the north pole
- the inside is free of field

[Tip for 4](#)

6. At which location of a current-carrying coil are the field lines densest?

- at the magnetic north pole
- at the magnetic south pole
- inside the coil
- at both poles

- Picture the two wires, or draw them on.
- In which direction would the outer field run in each case?
- The field is a linear vector field. So the total field can be created from several individual fields by adding them together. Does adding the field in between make it larger, or smaller?

- First imagine the parallel wires again. What happens when the current flows in the same direction and what happens when the current flows in opposite directions? Are the resulting forces equal in magnitude?
- The reversal of the direction of the current can now also be produced by turning the wire instead of changing the current - just so that the wires are perpendicular to each other in the meantime when turning.
- With parallel wires and different current directions, the amount-wise same force arises. So, this is also true for every angle in between (in detail given by integration of the force over single wire pieces).
- But then there must be a point at which the force becomes 0.

Check Answers

You Scored % - /

[Tip for 5](#)

- The magnetic field lines must be closed.
- Compare the field curve between the coil and permanent magnet.

[Tip for 6](#)

- In video 1 you can see the course outside and inside the coil.

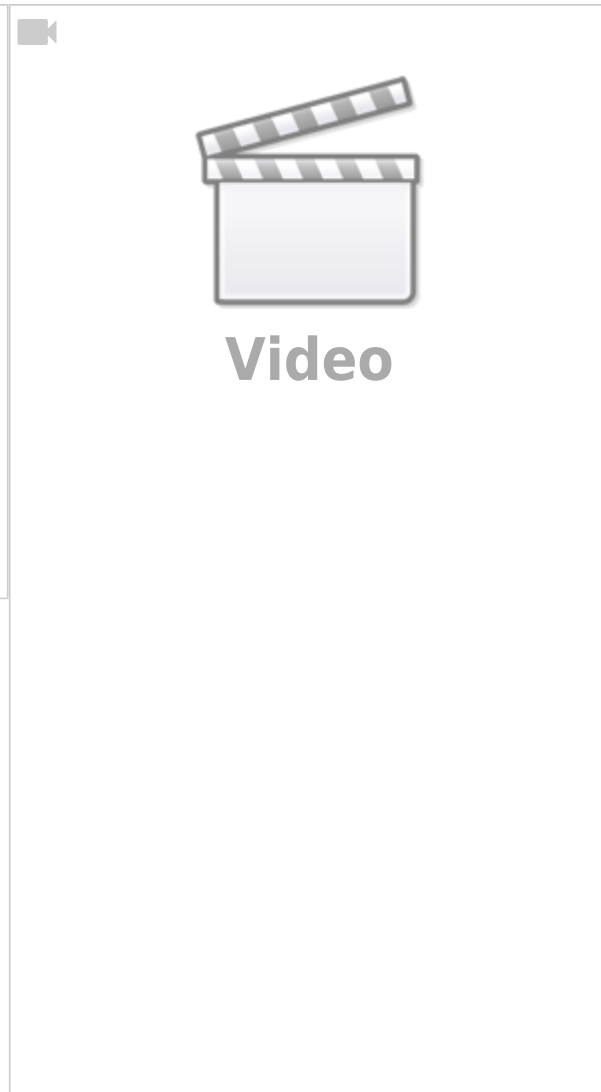
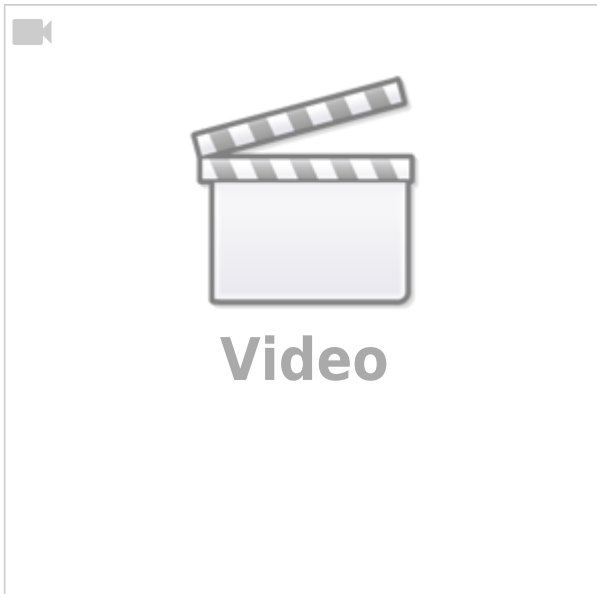
Embedded resources

Please have a look at the German contents (text, videos, exercises) on the page of the [KIT-Brückenkurs >> Lorentz-Kraft](#). The last part “Magnetic field within matter” can be skipped.

A living insect (“diamagnet”) floats in a very strong magnetic field



Explanation of diamagnetism and paramagnetism



1)

ideally: infinite long; in reality much longer, than the distance between them

From:

<https://wiki.mexle.org/> - **MEXLE Wiki**

Permanent link:

https://wiki.mexle.org/electrical_engineering_and_electronics_1/block17?rev=1764637807

Last update: **2025/12/02 02:10**

