# Block 18 — Magnetic Flux and Induction

## Student Group

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# Block 18 — Magnetic Circuits and Inductance

## Learning objectives

After this 90-minute block, you can

- …

## Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

## 90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

## Conceptual overview

1. …

## Core content

For this and the following chapter the online Book 'DC Electrical Circuit Analysis - A Practical Approach' is strongly recommended as a reference. In detail this is chapter 10.3 Magnetic Circuits

In the previous chapters, we got accustomed to the magnetic field. During this path, some similarities from the magnetic field to the electric circuit appeared (see figure 1).

Fig. 1: Similarities magnetic Circuit vs electric Circuit

In this chapter, we will investigate how far we have come with such an analogy and where it can be practically applied.

## Basics for Linear Magnetic Circuits

For the upcoming calculations, the following assumptions are made

1. The relationship between $B$ and $H$ is linear: $B = \mu \cdot H$
   This is a good estimation when the magnetic field strength lays well below saturation
2. There is no stray field leaking out of the magnetic field conducting material.

3. The fields inside of airgaps are homogeneous. This is true for small air gaps.

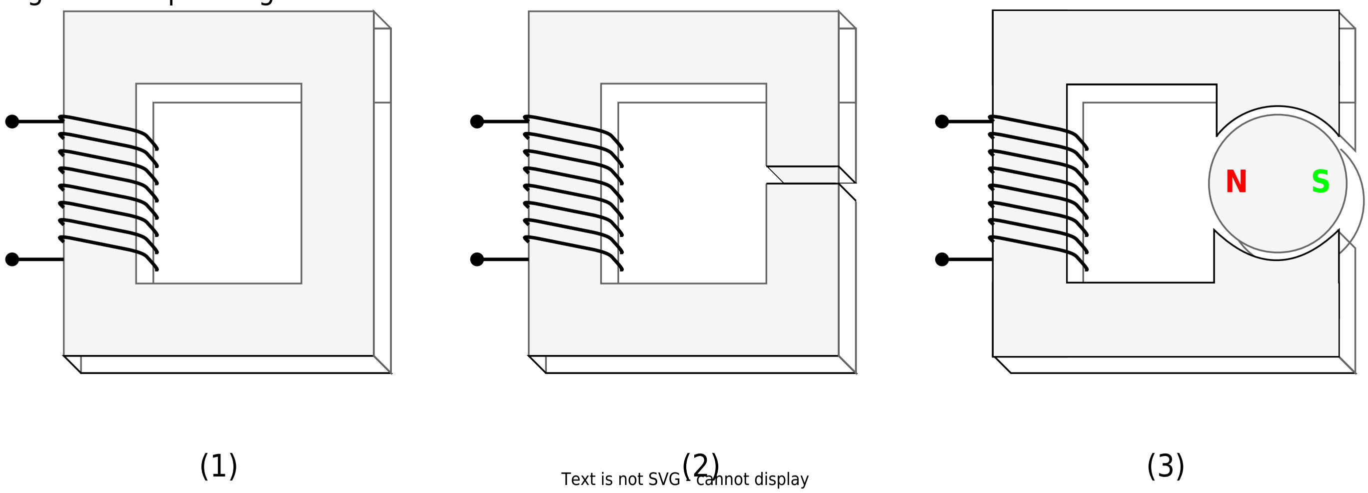
One can calculate a lot of simple magnetic circuits when these assumptions are applied and focusing on the average field line are applied.

Fig. 3: Simplifications and Linearization

Two simple magnetic circuits are shown in figure 2: They consist of

- a current-carrying coil
- a ferrite core
- an airgap (in picture (2) + (3) ).

Fig. 2: A simple magnetic Circuit




These three parts will be investigated shortly:

**Current-carrying Coil**

For the magnetic circuit, the coil is parameterized only by:

- its number of windings $N$ and
- the passing current $i$.

These parameters lead to the magnetic voltage $\theta = N\cdot i$.

**Ferrite Core**

- The core is assumed to be made of ferromagnetic material.
- Therefore, the relative permeability in the core is much larger than in air ($\mu_{\rm r,core}\gg 1$).
- The ferrite core is also filling the inside of the current-carrying coil.
- The ferrite core conducts the magnetic flux around the magnetic circuit (and by this: also to the airgap)

**Airgap**

- The air gap interrupts the ferrite core.
- The width of the air gap is small compared to the dimensions of the cross-section of the ferrite core.
- The field in the air gap can be used to generate (mechanical) effects within the air gap.
  An example of this can be the force onto a permanent magnet (see figure 2 (3)).

With the above-mentioned assumptions the magnetic flux $\Phi$ must remain constant along the ferrite core, so $\Phi_{\rm core}=\rm const.$
Since the magnetic field lines neither show sources nor sinks, also the flux passing

over to the airgap must be $\Phi_{\rm airgap}=\Phi_{\rm core}=\rm const.$ This can also be seen in figure ## (1) ).
A different view of this is the closed surface $\vec{A}$ (figure ## (2)): Based on the examination in Recap of magnetic Field we know that the flux into the volume must be equal the flux out of the volume, or $\Phi_{\rm m} = \iint_{A} \vec{B} \cdot d \vec{A} = 0$.

The relationship $B=\mu \cdot H$, and $\mu_{\rm core}\gg\mu_{\rm airgap}$ lead to the fact that the ${H}$-Field must be much stronger within the airgap (figure ## (3)).

Fig. ##: B- and H-field along the ferrite core

# Common pitfalls

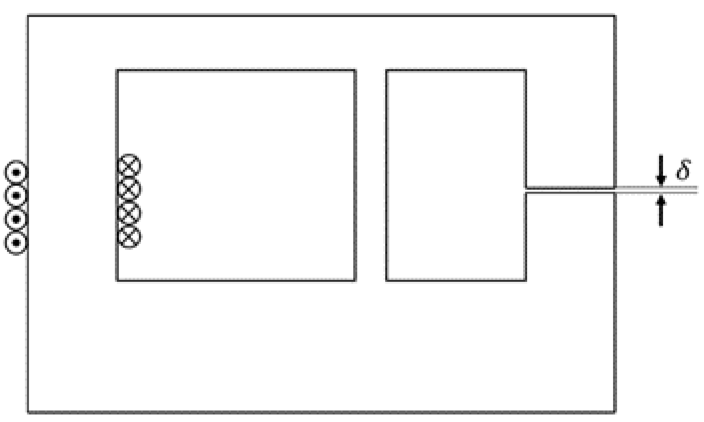
- …

# Exercises

**Exercise E1 Magnetic Circuit**
**(written test, approx. 7 % of a 120-minute written test, SS2022)**

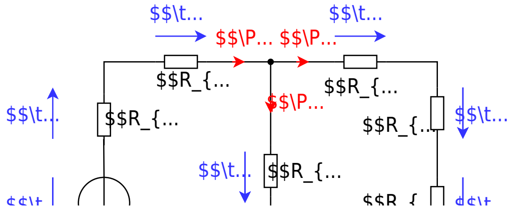
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages.
The components shall be designed in such a way, that the magnetic resistance is constant in it.
Formulas are not necessary.

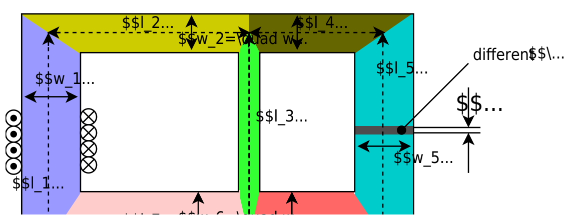
Result:




Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each “diversion”. \begin{align*} R_{\rm m} = {{1}\over{\mu_0 \boldsymbol{\mu_{\rm r}}} }{\boldsymbol{l}\over{\boldsymbol{w}\cdot h}} \end{align*}



**Exercise E16 Magnetic Circuit**
**(written test, approx. 9 % of a 120-minute written test, SS2024)**

A toroidal core (ferrite, $\mu_{\rm r}=900$) has a cross-sectional area of $A=300 ~\rm mm^2$ and an average circumference of $l=3 ~\rm dm$.

2. Calculate the resulting magnetic flux in the core.

Result

Path

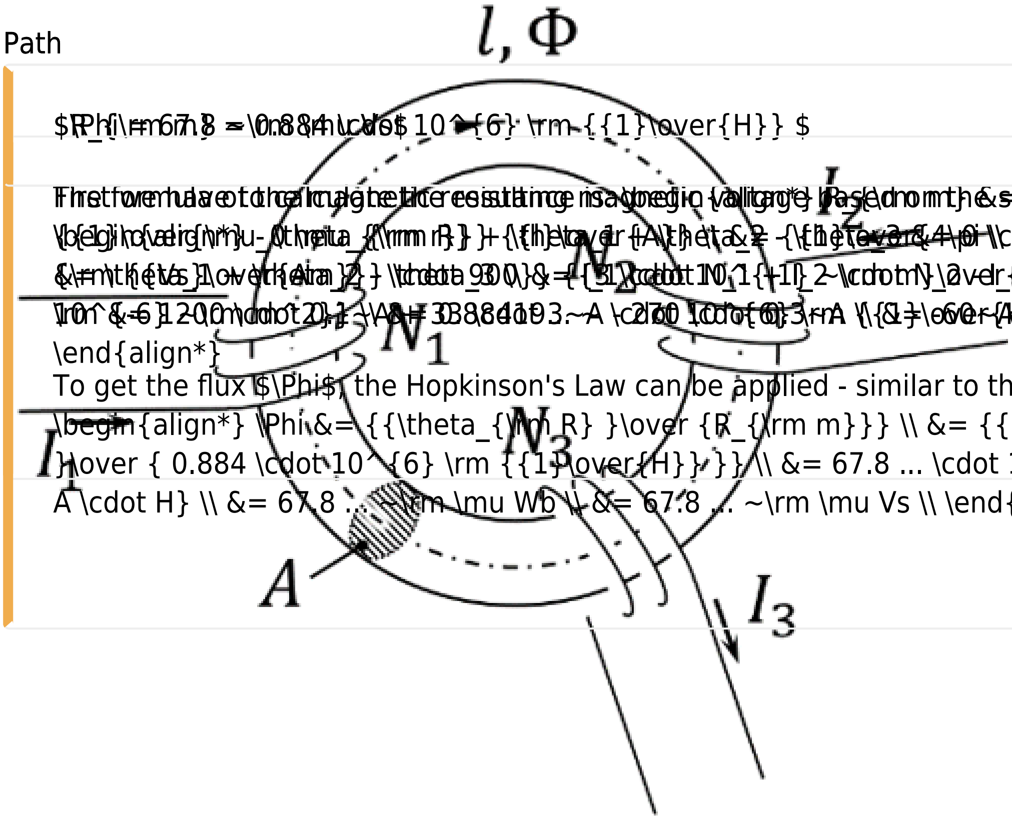
$\Phi = 67.8 ~\rm \mu Vs$

$R_{\rm m} = 0.884 \cdot 10^{6} \rm {{1}\over{H}} $

First we have to calculate the resulting magnetic voltage based on the sources: [illegible] &= -60~\rm A \end{align*}

The formula of the magnetic resistance is \begin{align*} R_{\rm m} &= [illegible] 10^{-7} [illegible] 3 \cdot 10^{-1} [illegible] 300 \cdot 10^{-6} [illegible] &= 0.884 \cdot 10^{6} \rm {{1}\over{H}} \end{align*}

To get the flux $\Phi$, the Hopkinson's Law can be applied - similar to the Ohm's Law:
\begin{align*} \Phi &= {{\theta_{\rm R} }\over {R_{\rm m}}} \\ &= {{-60~\rm A \over { 0.884 \cdot 10^{6} \rm {{1}\over{H}} }} \\ &= 67.8 ... \cdot 10^{-6} { \rm A \cdot H} \\ &= 67.8 ... ~\rm \mu Wb \\ &= 67.8 ... ~\rm \mu Vs \\ \end{align*}




On the core, there are three coils with:

- Coil 1: $N_1 = 1200$, $I_1=100 ~\rm mA$
- Coil 2: $N_2 = 33 $, $I_2= 3 ~\rm A$
- Coil 3: $N_3 = 270 $, $I_3=0.3 ~\rm A$

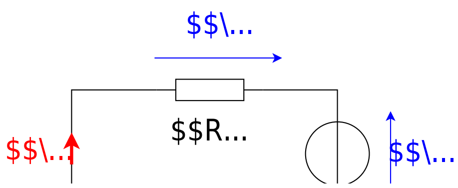
Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup.
Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single $R_\rm m$.
- For the orientation of the magnetic voltages $\theta_1$, $\theta_2$, and $\theta_3$, the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux $\Phi$
- The magnetic voltages are antiparallel to the flux for sources and parallel for the

load.




**Exercise E7 Cylindrical Coil**
**(written test, approx. 6 % of a 120-minute written test, SS2021)**

A cylindrical coil with the following information is given:

- Length $ = 30 {~\rm cm}$,
- Winding diameter $ = 390 {~\rm mm}$,
- Number of windings $ = 240$ ,
- Current in the conductor $ = 500 {~\rm mA}$,
- Material inside: Air
- $\mu_0 = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

a) the magnetic field strength (2 points)

**b) the magnetic flux density (2 points)**

Result

$H = 400 ~\rm A/m$

$B = 0.50 ~\rm mT$

Path

The magnetic flux density is given as $B = \mu_0 \mu_{\rm r} \cdot H$:

\begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 400 ~\rm {{A}\over{m}} \\ &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \end{align*}

**c) the magnetic flux (2 points)**

Result

$\Phi = ... ~\rm Wb$

Path

The magnetic flux is given as:

\begin{align*} \Phi &= B \cdot A \end{align*}

Since the coil is cylindrical, the cross-sectional area is given as:

\begin{align*} A = \pi r^2 = \pi \left( {{d}\over{2}} \right)^2 \end{align*}

Therefore: \begin{align*} \Phi &= B \cdot \pi \left( {{d}\over{2}} \right)^2

\end{align*}

Putting in the numbers: \begin{align*} \Phi &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \cdot \pi \left( {{0.39{\rm m}}\over{2}} \right)^2 \\ &= 0.00006004... {\rm Vs} \end{align*}

\begin{align*} H &= {{N \cdot I}\over{l}} = {{w \cdot 4}\over{l}} \end{align*}

Putting in the numbers: \begin{align*} H &= {{240 \cdot 0.5 {~\rm A}}\over{0.3 {~\rm m}}} \end{align*}

## Embedded resources

Explanation (video): …