Block 18 — Magnetic Flux and Induction

Student Group

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Block 18 — Magnetic Flux and Inductivity

Learning objectives

After this 90-minute block, you can

- …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Moving single Charge in a magnetic Field

Instead of a current in the magnetic field, we will now have a look on a charge moving in the magnetic field.
Since the electical current I is based on a moving charge ($I = {{dQ}\over{dt}}$), also a moving charge has to feel a force.
To find this force the force onto a conductor can be used as a start (see block17). However, the formula will be investigated infinitesimally for small parts ${\rm d} \vec{l}$ of the conductor:

$$\begin{align*} \vec{{\rm d}F}_{\rm L} = I \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}$$

The current is now substituted by $I = {\rm d}Q/{\rm d}t$, where ${\rm d}Q$ is the small charge packet in the length $\vec{{\rm d}l}$ of the conductor.

$$\begin{align*} \vec{{\rm d}F}_{\rm L} = {{{\rm d}Q}\over{{\rm d}t}} \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}$$

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

$$\begin{align*} \vec{{\rm d}F}_{\rm L} &= {{{\rm d}Q \cdot {\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ \end{align*}$$

Here, the part ${{{\rm d}\vec{l}}\over{{\rm d}t}}$ represents the speed $\vec{v}$ of the small charge packet ${\rm d}Q$.

$$\begin{align*} \vec{{\rm d}F}_{\rm L} &= {\rm d}Q \cdot \vec{v} \times \vec{B} \end{align*}$$

The **Lorenz Force** on a finite charge packet is the integration:

$$\begin{align*} \boxed{\vec{F}_{\rm L} = Q \cdot \vec{v} \times \vec{B}} \end{align*}$$

Notice:

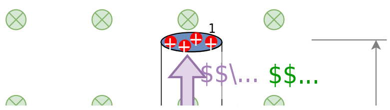
- A charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force of $\vec{F_{\rm L}}$.
- The direction of the force is given by the right-hand rule.
- The true Lorentz force is not the force on the whole conductor but the single force onto an (elementary) charge.

Moving single Rod in a magnetic Field

Coming from a single free charge, let us have a look onto free charges in a conductor.
These are also free to move within the borders of the conducting material.
The first step to investigate the motional induction is shown in figure 6: a single conducting rod with the length $\vec{l}$ which is moving with a constant velocity $\vec{v}$ through a homogenous magnetic field $\vec{B} \perp \vec{v} \perp \vec{l}$.

- The charges in the rod experience the Lorentz force $\vec{F}_{\rm L}$.
- By this force, the positive charges move to one end of the rod and the negative to the other one.
- The separated charges create a potential difference and by this, a Coulomb force $\vec{F}_{\rm C}$ onto the charges within the rod.
- For a constant speed, the Lorentz force onto charges in the rod must have the same magnitude as the Coulomb force.

Fig. 6: motional Induction on a single Rod



This leads to:

\begin{align*} \vec{F}_{\rm C} &= - \vec{F}_{\rm L} \\ Q \cdot \vec{E}_{\rm ind} &= - Q \cdot \vec{v} \times \vec{B} \\ \vec{E}_{\rm ind} &= - \vec{v} \times \vec{B} \\ \end{align*}

The induced potential difference in the rod will be:

\begin{align*} u_{\rm ind} &= \int_l \vec{E}_{\rm ind} \cdot {\rm d} \vec{s} \\ &= - \int^0_1 \vec{v} \times \vec{B} \cdot {\rm d} \vec{s} \\ \end{align*}

For constant $|\vec{v}|$ and $|\vec{B}|$ this leads to: \begin{align*} u_{\rm ind} &= - v \cdot B \cdot l \\ \end{align*}

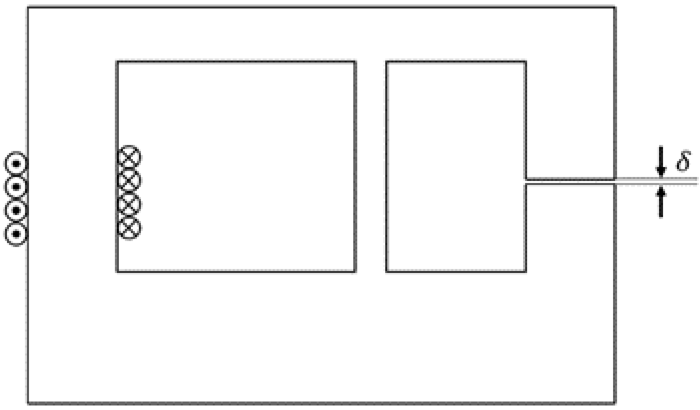
Common pitfalls

- ...

Exercises

Exercise E10 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages.
The components shall be designed in such a way, that the magnetic resistance is constant in it.
Formulas are not necessary.

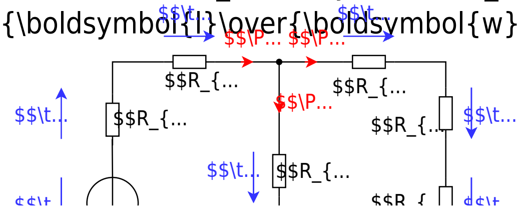


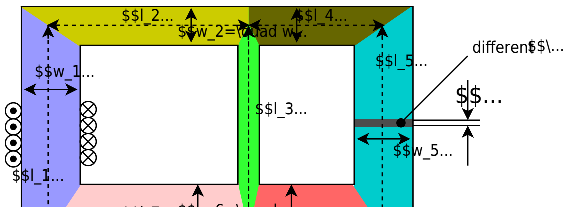
Result

Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each “diversion”.

$$R_{\rm m} = {{1}\over{\mu_0 \boldsymbol{\mu_{\rm r}}} }{\boldsymbol{l}\over{\boldsymbol{w}\cdot h}}$$





Exercise E10 Magnetic Circuit
(written test, approx. 9 % of a 120-minute written test, SS2024)

A toroidal core (relative permeability $\mu_r=900$) with a cross-sectional area of $A=300 ~\rm mm^2$ and an average circumference of $l=3 ~\rm dm$.

2. Calculate the resulting magnetic flux in the core.

Result

Path

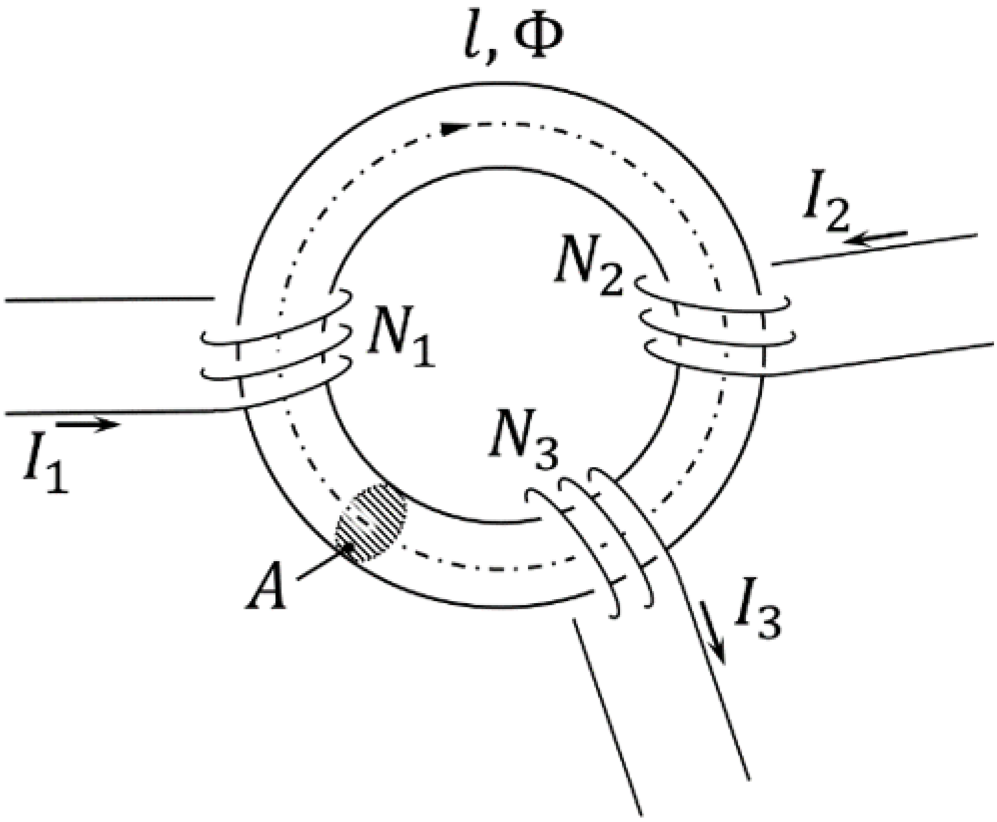
$R_{\rm m} = 0.884 \cdot 10^{6} \rm {{1}\over{H}} $

$\Phi = 67.8 ~\rm \mu Vs$

The formula of the magnetic resistance is: \begin{align*} R_{\rm m} &= {{1}\over{\mu_0 \mu_r}}\cdot {{l}\over{A}} \\ &= {{1}\over{4\pi \cdot 10^{-7} {\rm Vs}\over{\rm Am}} \cdot 900} \cdot {{3 \cdot 10^{-1} ~\rm m}\over{300 \cdot 10^{-6} ~\rm m^2}} \\ &= 0.884 \cdot 10^{6} \rm {{1}\over{H}} \end{align*}

First we have to calculate the resulting magnetic voltage based on the sources: \begin{align*} \theta_{\rm R} &= \theta_1 + \theta_2 - \theta_3 \\ &= ... \\ &= -60~\rm A \\ \end{align*}

To get the flux Φ, the Hopkinson's Law can be applied - similar to the Ohm's Law: \begin{align*} \Phi &= {{\theta_{\rm R} }\over {R_{\rm m}}} \\ &= {{-60~\rm A }\over { 0.884 \cdot 10^{6} \rm {{1}\over{H}} }} \\ &= 67.8 ... \cdot 10^{-6} { \rm A \cdot H} \\ &= 67.8 ... ~\rm \mu Wb \\ &= 67.8 ... ~\rm \mu Vs \\ \end{align*}



On the core, there are three coils with:

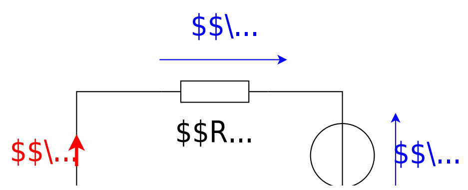
- Coil 1: $N_1 = 1200$, $I_1=100 ~\rm mA$
- Coil 2: $N_2 = 33 $, $I_2= 3 ~\rm A$
- Coil 3: $N_3 = 270 $, $I_3=0.3 ~\rm A$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup.
Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single $R_\rm m$.
- For the orientation of the magnetic voltages θ_1, θ_2, and θ_3, the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E1 Cylindrical Coil
(written test, approx. 6 % of a 120-minute written test, SS2021)

A cylindrical coil with the following information is given:

b) the magnetic flux density (2 points)

c) the magnetic flux (2 points)

Result

- Length l = 30 {~\rm cm}$,
- Winding diameter d = 390 {~\rm mm}$,
- Number of windings N = 240$,
- Current in the conductor I = 500 {~\rm mA}$,
- Material inside: Air
- $\mu_0 = 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}}$

Path

$H = 400 ~\rm A/m$

$B = 0.50 ~\rm mT$

$\Phi = 60 ~\rm µWb$

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

a) the magnetic field strength (2 points)

The magnetic field strength is

The magnetic flux density is given as $B = \mu_0 \mu_{\rm r} \cdot H$:

$$\Phi = B \cdot A$$

$$B = \mu_0 \cdot H$$

Since the coil is cylindrical, the cross-sectional area is given as

Putting in the numbers:
$$\begin{align*} B &= 4\pi\cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \cdot 400 ~\rm {{A}\over{m}} \\ &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \end{align*}$$

$$A = \pi r^2 = \pi \left({{d}\over{2}} \right)^2$$

Path

Therefore:
$$\Phi = B \cdot \pi \left({{d}\over{2}} \right)^2$$

Putting in the numbers:
$$\begin{align*} \Phi &= 0.0005026... {{\rm Vs}\over{\rm m^2}} \cdot \pi \left({{0.39{\rm m}}\over{2}} \right)^2 \\ &= 0.00006004... {\rm Vs} \end{align*}$$

$$\begin{align*} H &= {{N \cdot I}\over{l}} = {{w \cdot I}\over{l}} \end{align*}$$

Putting in the numbers:
$$\begin{align*} H &= {{240 \cdot 0.5 {~\rm A}}\over{0.3 {~\rm m}}} \end{align*}$$

Embedded resources

Explanation (video): …