

Block 20 — Inductance and Energy

Student Group

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Block 20 — Inductance and Energy

20.0 Intro

20.0.1 Learning objectives

After this 90-minute block, you can

- ...

20.0.2 Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

20.0.3 90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

20.0.4 Conceptual overview

1. ...

20.1 Core content

20.1.1 Self-Induction

Up to now, we investigated the induction of electric voltages and currents based on the change of an external flux $\frac{d\psi}{dt}$. For the induced current i_{ind} , we found that it counteracts the change of the external flux (Lenz law).

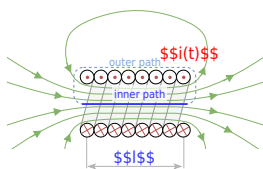
But what happens, when there is no external field - only a coil which creates the flux change itself

(see [figure 1](#))?

Fig. 1: Induction Phenomenons

To understand this, we will investigate the situation for a long coil (figure 2).

Fig. 2: Self-Induction of a Coil



Given by the [Recap of the fieldline images](#) in Block16, we know that the H -field is given by magnetic voltage $\theta(t) = N \cdot i$ as:
$$H(t) = \frac{N \cdot i}{l}$$

This lead to the magnetic flux density $B(t)$

$$B(t) = \mu_0 \mu_r \cdot \frac{N \cdot i}{l}$$

Based on the magnetic flux density $B(t)$ it is possible to calculate the flux $\Phi(t)$:

$$\Phi(t) = \int_A \vec{B}(t) \cdot d\vec{A} = \int_A \mu_0 \mu_r \cdot \frac{N \cdot i}{l} \cdot dA = \mu_0 \mu_r \cdot \frac{N \cdot i}{l} \cdot A$$

The changing flux Φ is now creating an induced electric voltage and current, which counteracts the initial change of the current.

This effect is called **Self Induction**. The induced electric voltage u_{ind} is given by:

$$u_{\text{ind}} = -N \cdot \frac{d\Phi(t)}{dt} = -N \cdot \frac{d(\mu_0 \mu_r \cdot \frac{N \cdot i}{l} \cdot A)}{dt} = -N \cdot \mu_0 \mu_r \cdot \frac{N \cdot A}{l} \cdot \frac{di}{dt}$$

$$\boxed{u_{\text{ind}}} = -\mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} \cdot \frac{di}{dt}$$

$$\frac{d\psi}{dt} \quad \text{\textit{for a long coil}}$$

The result means that the induced electric voltage u_{ind} is proportional to the change of the current $\frac{d\psi}{dt}$.

The proportionality factor is also called **Self-inductance** L (or often simply called inductance).

20.1.2 Inductance

The inductance is another passive basic component of the electric circuit. Besides the ohmic resistor R and the capacitor C , the inductor L is the lump component entailing the inductance.

Generally, the inductance is defined by:
$$L = \frac{u_{\text{ind}}}{\frac{d\psi}{dt}}$$

The inductance L can also be described differently based on Lenz law $u_{\text{ind}} = - \frac{d\psi(t)}{dt}$:

$$L = \frac{u_{\text{ind}}}{\frac{d\psi(t)}{dt}} = \frac{d\psi(t)/dt}{\frac{d\psi(t)}{dt}}$$

$$L = \frac{\psi(t)}{i}$$

One can also consider an inductor a “conservative person”: it does not like to see abrupt changes in the passing current. It reacts to any change in the current with a counteracting voltage since the current change leads to a changing flux and - therefore - an induced voltage. The [figure 3](#) shows an inductor in series with a resistor and a switch (any real switch also behaves as a capacitor, when open). Once the simulation is started, the inductor directly counteracts the current, which is why the current only slowly increases.

The unit of the inductance is $1 \text{ Henry} = 1 \text{ H} = 1 \frac{\text{Vs}}{\text{A}} = 1 \frac{\text{Wb}}{\text{A}}$

Fig. 3: Example of a Circuit with an Inductor

Mathematically the voltages can be described in the following way:

$$u_0 = u_R + u_L \quad u_L = i \cdot R + \frac{d\psi}{dt} = i \cdot R + L \cdot \frac{di}{dt}$$

20.1.3 Inductance of different Components

Long Coil

In the last sub-chapter, the formula of a long coil was already investigated. By these, the inductance of a long coil is

$$L_{\text{long coil}} = \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l}$$

Toroidal Coil

The toroidal coil was analyzed in the last chapter (see [magnetic Field Strength Part 1: Toroidal Coil](#)). Here, a rectangular intersection is assumed (see [figure 4](#)).

Fig. 4: Self-Induction of a toroidal Coil

This leads to

$$H(t) = \frac{N \cdot i}{l}$$

with the mean magnetic path length (= length of the average field line) $l = \pi(r_o + r_i)$:

$$H(t) = \frac{N \cdot i}{\pi(r_o + r_i)}$$

The inductance L can be calculated by

$$L_{\text{toroidal coil}} = \frac{\Psi(t)}{i} = \frac{N \cdot \Phi(t)}{i}$$

With the magnetic flux density $B(t) = \mu_0 \mu_r H(t) = \mu_0 \mu_r \frac{N \cdot i}{l}$ and the cross section $A = \pi(r_o - r_i)$, we get:

$$\begin{aligned} \text{\quad \quad } L_{\text{\rm toroidal \; coil}} &= \frac{N \cdot \mu_0 \mu_{\text{\rm r}} \cdot \{i \cdot N\}}{\pi(r_{\text{\rm o}} + r_{\text{\rm i}})} \cdot \frac{h(r_{\text{\rm o}} - r_{\text{\rm i}})}{i} \quad \&= \frac{N^2 \cdot \mu_0 \mu_{\text{\rm r}} \cdot h(r_{\text{\rm o}} - r_{\text{\rm i}})}{\pi(r_{\text{\rm o}} + r_{\text{\rm i}})} \end{aligned}$$

$$\boxed{L_{\text{\rm toroidal \; coil}}} = \mu_0 \mu_{\text{\rm r}} \cdot N^2 \cdot \frac{h(r_{\text{\rm o}} - r_{\text{\rm i}})}{\pi(r_{\text{\rm o}} + r_{\text{\rm i}})}$$

20.1.4 Inductances in Circuits

Focus here: uncoupled inductors!

Series Circuits

Based on $L = \frac{\Psi(t)}{i}$ and Kirchhoff's mesh law ($i = \text{const}$) the series circuit of inductions can be interpreted as a single current i which generates multiple linked fluxes Ψ . Since the current must stay constant in the series circuit, the following applies for the equivalent inductor of a series connection of single ones:

$$L_{\text{\rm eq}} = \frac{\sum_i \Psi_i}{i} = \sum_i L_i$$

A similar result can be derived from the induced voltage $u_{\text{\rm ind}} = L \frac{di}{dt}$, when taking the situation of a series circuit (i.e. $i_1 = i_2 = i_3 = \dots = i_{\text{\rm eq}}$ and $u_{\text{\rm eq}} = u_1 + u_2 + \dots$):

$$\begin{aligned} u_{\text{\rm eq}} &= u_1 + u_2 + \dots \quad \& L_{\text{\rm eq}} \frac{di_{\text{\rm eq}}}{dt} \\ &= L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + \dots \\ &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots \end{aligned}$$

Parallel Circuits

For parallel circuits, one can also start with the principles based on Kirchhoff's mesh law:

$$u_{\text{\rm eq}} = u_1 = u_2 = \dots$$

and Kirchhoff's nodal law:

$$i_{\text{\rm eq}} = i_1 + i_2 + \dots$$

Here, the formula for the induced voltage has to be rearranged:

$$u_{\text{\rm ind}} = L \frac{di}{dt} \quad \& \int u_{\text{\rm ind}} dt = L \cdot i \quad \& i = \frac{1}{L} \int u_{\text{\rm ind}} dt$$

By this, we get:

$$\begin{aligned} i_{\text{\rm eq}} &= i_1 + i_2 + \dots \quad \& \frac{1}{L_{\text{\rm eq}}} \int u_{\text{\rm eq}} dt \\ &= \frac{1}{L_1} \int u_1 dt + \frac{1}{L_2} \int u_2 dt + \dots \\ &= \frac{1}{L_{\text{\rm eq}}} \int u dt = \frac{1}{L_1} \int u dt + \frac{1}{L_2} \int u dt + \dots \end{aligned}$$

Notice:

The inductor behaves in the parallel and series circuit similar to the resistor.

20.1.5 Energy of the magnetic Field

not covered

20.2 Common pitfalls

- ...

20.3 Exercises

Exercise E1 Self-Induction

(written test, approx. 8 % of a 120-minute written test, SS2024)

A coil with a length of 0.30 m carries a current of 3 A and has 500 turns. The current through the coil changes linearly from 0 A to 3 A in 0.02 ms . The arrangement is located in air ($\mu_{\text{r}}=1$).

Path

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$$

$$U_{\text{ind}} = \dots \text{ V}$$

.. Calculate the (self-)inductance of the coil.

For the linear change of the current the formula of the induced voltage can also be linearized:
$$u_{\text{ind}} = -L \cdot \frac{di}{dt}$$

$$\rightarrow -L \cdot \frac{\Delta i}{\Delta t} = -1.32 \cdot 10^{-3} \cdot \frac{3 \text{ A}}{0.02 \cdot 10^{-3} \text{ s}}$$

The formula for the induction of a long coil is:
$$L = \mu_0 \mu_{\text{r}} \cdot N^2 \cdot \frac{A}{l} = 4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot (500)^2 \cdot \frac{\pi \cdot (2 \cdot 10^{-2} \text{ m})^2}{2 \cdot 10^{-2} \text{ m}}$$

Exercise E12 Self Induction

(written test, approx. 8 % of a 120-minute written test, SS2022)

A circuit with a resistor and an inductor is connected to a DC voltage source which is fused with a circuit breaker.

$$\cdot 10^{-7} \frac{\text{H}}{\text{m}} \cdot 1 \cdot (390)^2 \cdot \frac{(\pi \cdot (0.03 \text{ m})^2)^2}{0.18 \text{ m}} \end{align*}$$

Exercise 4.5.2 Self Induction II

A cylindrical air coil (length $l=40 \text{ cm}$, radius $r=5.0 \text{ cm}$, and a number of windings $N=300$) passes a current of 30 A . The current shall be reduced linearly in 2.0 ms down to 0.0 A .

What is the amount of the induced voltage u_{ind} ?

$$\begin{align*} |u_{\text{ind}}| &= 33 \text{ V} \end{align*}$$

Solution

The requested induced voltage can be derived by:

$$\begin{align*} L &= \left| \frac{u_{\text{ind}}}{\text{d}i / \text{d}t} \right| \\ \rightarrow |u_{\text{ind}}| &= L \cdot \left| \frac{\text{d}i}{\text{d}t} \right| \\ &= L \cdot \left| \frac{\Delta i}{\Delta t} \right| \end{align*}$$

Therefore, we just need the inductance L , since $\frac{\Delta i}{\Delta t}$ is defined as 30 A per 2 ms :

$$\begin{align*} L &= \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} \\ \end{align*}$$

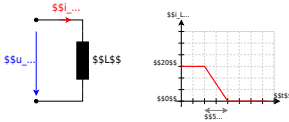
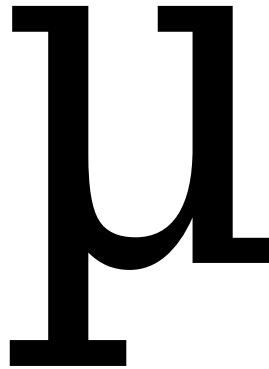
$$\begin{align*} |u_{\text{ind}}| &= \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} \cdot \left| \frac{\Delta i}{\Delta t} \right| \\ &= 4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \cdot 1 \cdot (300)^2 \cdot \frac{\pi \cdot (0.05 \text{ m})^2}{0.40 \text{ m}} \cdot \frac{30 \text{ A}}{2 \text{ ms}} \end{align*}$$

Exercise 4.5.3 Self Induction III

A coil with the inductance $L=20 \text{ }\mu\text{H}$ passes a current of 40 A . The current shall be reduced linearly in $5 \text{ }\mu\text{s}$ down to 0 A (see [figure 5](#)).

- What is the amount of the induced voltage u_{ind} ?
- Sketch the course of $u_{\text{ind}}(t)$!

Fig. 5: Circuit and timing Diagram



Embedded resources

Explanation (video): ...

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Last update: **2026/01/20 15:39**

