Block 22 — Negative-feedback Op-Amp Circuits

Student Group

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Table of Contents

Block 22 — Negative-feedback Op-Amp Circuits

Learning objectives

After this 90-minute block, you can

1. … apply the superposition method to operational amplifier circuits.
2. … know the circuit and transfer function of a voltage-to-current converter and current-to-voltage converter look like.
3. … name applications for the summing inverter, voltage-to-current converter, and current-to-voltage converter.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Introductory Example

In various applications, currents must be measured. In an electric motor, for example, the torque is caused by the current flowing through the motor. A motor control and a simple overcurrent shutdown are based on the knowledge of the current. For further processing, a voltage must be generated from the current. The simplest current-to-voltage converter is the ohmic resistor. A sufficiently large voltage as required by a microcontroller, for example, cannot be achieved with this. So not only does the current have to be converted, but also the generated potential difference has to be amplified.

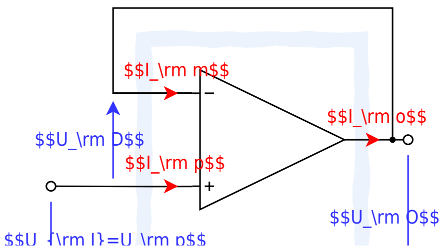
One such current sense amplifier is the INA 240 device. This is installed as shown below. In the simulation, a real current source feeds the electrotechnical image of a DC motor on the left (in the example: inductance with $L_{\rm L}=10~\rm mH$ and internal resistance $R_{\rm L}=1~\Omega$). The current flowing from the motor is conducted through a measuring resistor ($R_{~\rm M}=0.01~\Omega$) which is noticeably smaller than the internal resistance of the motor. Thus, most of the power acts in the motor and the current is only marginally affected by the sense resistor. The simulation above shows the inner workings of the current measuring amplifier.

The following explains ways in which such circuits can be understood.

Voltage follower

In the Block21 it was described that an amplifier with high open-loop gain can be “tamed” by feeding back a part of the output signal with a negative sign.
In the simplest case, the output signal could be fed directly to the negative input of the operational amplifier. The input signal $U_\rm I$ of the entire circuit is applied to the positive input. In figure 1 this circuit is shown.

Fig. 1: voltage follower



Using this circuit, the procedure for solving amplifier circuits is now to be illustrated.

1. The aim is always to create a relation between output voltage $U_\rm O$ and input voltage $U_\rm I$.
   Thus, the goal here is the voltage gain $A_{\rm V}=\frac{U_{\rm O}}{U_\rm I}$.

2. Before calculating, it should be checked how many equations describe the system and thus have to be set up.
   This can be determined by the **number of variables**. This is done by counting through the currents and voltages of the circuit.
   In this case, there are 3 currents and 3 voltages. So the **number of equations** needed is 6.

3. Now **equations are set up** that can be used. These are:
   1. **Basic equation**: (1) $U_{\rm O} = U_{\rm D} \cdot A_\rm D$
   2. **Golden rules**: $R_\rm D \rightarrow \infty$ so that (2+3) $I_{\rm p} = I_{\rm m} = 0$, $A_{\rm D} \rightarrow \infty$, $R_\rm O = 0$

    3. Consideration of the existing **loops**: in this example, there is only one loop (4) $-U_{\rm I} + U_{\rm D} + U_{\rm O} = 0$.
       Caution: loops can not enter the amplifier through input and exit through the output! Also to be noted is the direction of $U_{\rm D}$.
    4. Consideration of the existing **nodes**: in this example, there is only one node (5) $I_{\rm o} = I_m$.

4. There appears to be a missing equation. However, this is not correct, because there is still an equation hidden in the objective: (0) $A_{\rm V}=\frac{U_{\rm O}}{U_\rm I}$

5. Now, to **solve the equations**, the equations must be cleverly inserted into each other in such a way that there are no dependencies on the variables left at the end.

The calculation is done here once in detail (clicking on the arrow to the right "►" leads to the next step, alternative representation):

So the voltage gain is $A_\rm V = 1$. This would also have been seen in chapter Block21. There it was derived that for $A_\rm D \rightarrow \infty$ the voltage gain just results from k: $A_{\rm V}=\frac{1}{k}$.
Since the entire output voltage is fed back here, $k=1$ and thus also $A_\rm V=1$.

The output voltage $U_\rm O$ is therefore equal to the input voltage $U_\rm I$. This is where the name "voltage follower" comes from. Now one could assume, that this amplifier is of little help because also a direct connection would deliver $U_{\rm O}=U_{\rm I}$.
But the important thing here is: because of the operational amplifier, there is no feedback from U_O to U_I.
This means, that a resistor on the output side will not load the input side. In the simulation, the "Resistance" slider (on the right) can be used to change the load resistance. This changes the current flow, but not the voltage.

This behavior can also be explained in another way: The input signal usually comes from a voltage source, which can only produce low currents.
That means the input signals are high impedance ($\text{high impedance}=\frac{\text{voltage}}{\text{low current}}$).
However, a load of arbitrary impedance can be applied to the output. That is, to keep the output signal constant, a large current must be provided depending on the load.
As the output resistance of the amplifier approaches 0, the signal is low impedance ($\text{low impedance}=\frac{\text{voltage}}{\text{(likely) large current}}$). This is where the second name of the circuit "**impedance converter**" comes from.

Remember: steps to the goal

To solve tasks, the following procedure helps:

1. Where to?
   Clarification of the goal (here: always the relation between output and input signal)

2. What to?
   Clarification of what is needed (here: always equations. The number of needed equations can be determined by the number of variables)

3. With what?
   Clarification of what is already available (here: known equations: voltage gain equation, basic equation, golden rules, loop/node theorem, relationships of voltages and currents of components).

4. Go.
   Work out the solution (here: inserting the equations) It helps to rearrange the equation so that $1/A_\rm D$ appears without a prefactor. It is valid: $1/A_{\rm D} \xrightarrow{A_{\rm D} \rightarrow \infty} 0$

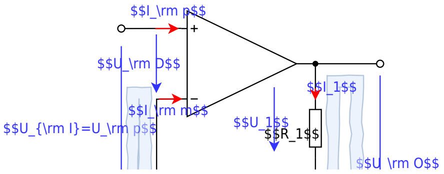
Non-inverting amplifier

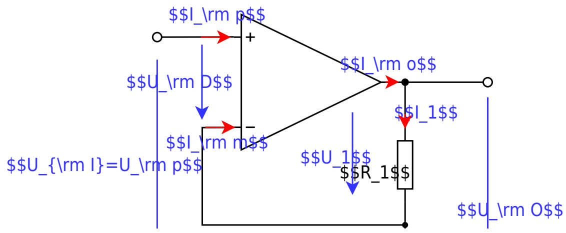
So far, the entire output voltage has been negative-feedback. Now only a part of the voltage is to be fed back.
To do this, the output voltage can be reduced using a voltage divider R_1+R_2. The circuit for this can be seen in figure 2.

By considering the feedback, the result can be quickly derived here as well: only $\frac{R_2}{R_1+R_2}\cdot U_{\rm O}$ is fed back from the output voltage $U_{\rm O}$.
So the feedback factor is $k=\frac{R_2}{R_1+R_2}$ and thus the voltage gain becomes $A_{\rm V}=\frac{R_1+R_2}{R_2}$.

This “trick” via $A_{\rm V}=\frac{1}{k}$ is no longer possible for some of the following circuits. Accordingly, a possible solution via network analysis is to be derived here as well.

Fig. 2: Non-inverting amplifier


$$I_\rm p$$
$$U_\rm D$$
$$I_\rm m$$
$$U_{\rm I}=U_\rm p$$
$$U_1$$
$$R_1$$
$$I_1$$
$$U_\rm O$$



Step	Description	Implementation
1	What is wanted?	$A_{\rm V} = \frac{U_{\rm O}}{U_{\rm I}}=?$
2	Counting the variables $\rightarrow$ Number of equations needed	5 voltages + 5 currents $\rightarrow$ Number of equations needed: 10

Step	Description	Implementation
3	Setting up the equations	always usable equations (1) Basic equation: $U_{\rm O} = A_{\rm D} \cdot U_{\rm D}$ Golden rules: $R_\rm D \rightarrow \infty$ so that (2+3) $I_\rm p \rightarrow 0$ and $I_\rm m \rightarrow 0$ $R_{\rm O} = 0$ $A_\rm D \rightarrow \infty$ Loops and nodes (see) (4) Loop I: $-U_{\rm I} + U_{\rm D} + U_2 = 0$ (5) Loop II: $-U_2 -U_1 + U_{\rm O} = 0$ (6) Node I: $I_{\rm o} = I_1$ (7) Node II / voltage divider: $I_1 - I_2 - I_\rm m = 0$ U, I relationships across components (8) Resistor $R_1= \frac{U_1}{I_1}$ (9) Resistor $R_2= \frac{U_2}{I_2}$

The calculation is done here again in detail (clicking the right arrow "►" leads to the next step, alternative representation):

So the voltage gain of the non-inverting amplifier is $A_{\rm V}=\frac{R_1+R_2}{R_2}$ or $A_{\rm V}=1+\frac{R_1}{R_2}$. Thus, the numerical value $A_{\rm V}$ can only become larger than 1.
This is shown again in the simulation. In real circuits, the resistors R_1 and R_2 will be in the range between a few $100 ~\Omega$ and a few $\rm M\Omega$.
If the sum of the resistors is too small, the operational amplifier will be heavily loaded. However, the output current must not exceed the maximum current.
If the sum of the resistors is too large, the current $I_1 = I_2$ can come into the range of the current $I_\rm m$, which is present in the real operational amplifier.

The **input and output resistance of the entire circuit** should also be considered here.
Both resistors are marked here with a superscript 0 to distinguish them from the input and output resistance of the operational amplifier.
The input resistance $R_{\rm I}^0$ is given by $R_{\rm I}^0=\frac{U_\rm I}{I_\rm I}$ with $I_{\rm I}=I_\rm p$. Thus, for the ideal operational amplifier, it is also true that the input resistance $R_{\rm I}^0=\frac{U_\rm I}{I_\rm p} \rightarrow \infty$ becomes when $I_\rm p \rightarrow 0$.

In the **real case** it is important in how far the total input resistance depends on the input resistance of the operational amplifier $R_{\rm I}^0(R_\rm D)$.
This can be derived as follows: (clicking on the right arrow "►" leads to the next step, alternative representation):

So it can be assumed simplistically, that the input resistance of the whole circuit is many times higher than the input resistance of the operational amplifier.

The output resistance $R_\rm O^0$ of the whole circuit with real operational amplifiers shall only be sketched: In this case, the output resistance $R_\rm O$ of the operational amplifier is in parallel with $R_1 + R_2$. Thus the output resistance $R_\rm O^0$ will be somewhat smaller than $R_\rm O$.

Notice: non-inverting amplifier

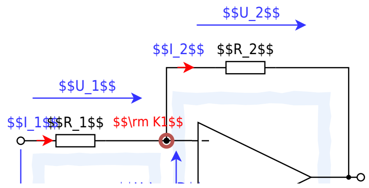
For the non-inverting amplifier, the following holds:

- The input voltage $U_\rm I$ is at the non-inverting input of the operational amplifier.
- The feedback is done by a voltage divider $R_1 + R_2$
- The voltage gain is $A_{\rm V}=\frac{R_1+R_2}{R_2}$ or $A_{\rm V}=1+\frac{R_1}{R_2}$ and is always greater than 1.
- Both input and output resistances of the overall circuit are smaller than those for the (real) operational amplifier used.

Inverting Amplifier

The circuit of the inverting amplifier can be derived from that of the non-inverting amplifier (see figure 4).
To do this, first consider the noninverting amplifier as a system with 3 connections (or as a voltage divider): $U_\rm I$, $\rm GND$, and $U_\rm O$.
These terminals can be rearranged - while keeping the output terminal $U_\rm O$.

Fig. 3: Inverting Amplifier



Thus, the voltage divider $R_1 + R_2$ is no longer between $U_\rm O$ and $\rm GND$, but between $U_\rm O$ and $U_\rm O$, see figure 3.
In this circuit, the resistor R_2 is also called the negative feedback resistor.

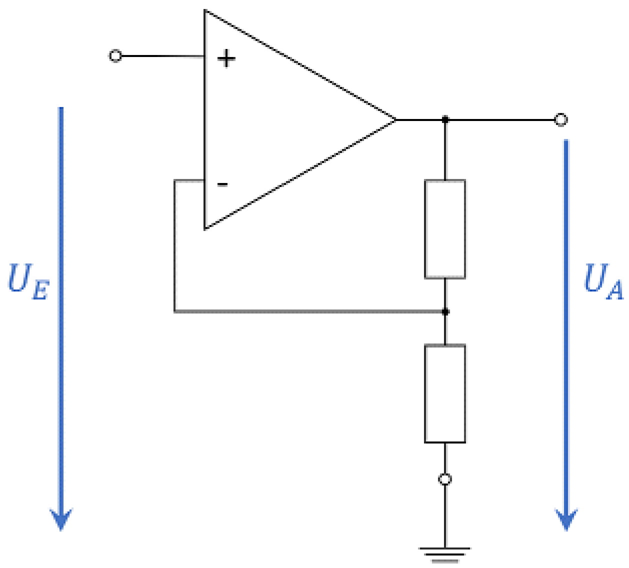


Fig. 4: Converting non-inverting amplifier to inverting amplifier. \\ $U_\rm E$ is the input voltage (Eingangsspannung), $U_\rm A$ is the output

voltage (Ausgangspannung).

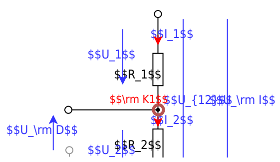
Before the voltage gain is determined, the node $\rm K1$ in figure 3 is to be considered first. This is just larger than the ground potential by the voltage $U_\rm D$; thus, it lies on the potential difference $U_\rm D$. For a feedback amplifier with finite voltage supply, $U_\rm O$ can only be finite, and thus $U_{\rm D}= U_{\rm O} / A_{\rm D} \rightarrow 0$, since $A_D \rightarrow \infty$ holds. Thus, it can be seen that the node $\rm K1$ is always at ground potential in the ideal operational amplifier. This property is called **virtual ground** because there is no direct short to ground. The op-amp regulates its output voltage $U_\rm O$ in such a way that the voltage divider sets a potential of $0~\rm V$ at node $\rm K1$. This can also be seen in the simulation by the voltage curve at $\rm K1$.

The following diagram shows again the interactive simulation.
$R_{\rm 1}$ and $R_{\rm 2}$ can be manipulated by the sliders. Hit Run / STOP to run

Notice: virtual ground

For the ideal feedback amplifier, $U_\rm D \rightarrow 0$ holds. This means that the same voltage is always present at both inputs.
If one of the two voltages is fixed, for example by connecting ground potential or even by a fixed voltage source, this property is called **virtual ground**.

Fig. 5: Voltage divider in inverting amplifier



For the determination of the voltage gain, the consideration of the feedback $A_{\rm V}=\frac{1}{k}$ seems to be of little use at first. Instead, however, the determination via network analysis is possible. figure 3 shows a possible variant to choose the loops for this purpose. However, network analysis is not to be done here, but is given in Exercise 3.5.1 below.

Instead, two other ways of derivation will be shown here to bring further approaches closer. For the first derivation, the **voltage divider** $\boldsymbol{R_1 + R_2}$ is considered. For the unloaded voltage divider, the general rule is:

$$\begin{align*} U_2 = U_{12} \cdot \frac{R_2}{R_1 + R_2} \end{align*}$$

This equation is now to be adapted for concrete use. First figure 3, the voltages of the voltage divider can be read as given in figure 5. From this, using the general voltage divider formula:

$$\begin{align*} U_2 = (U_I - U_O) \cdot \frac{R_2}{R_1 + R_2} \end{align*}$$

With the virtual mass at node $\rm K1$ in figure 5, it holds that U_2 points away from the (virtual) mass and thus $U_2 = U_\rm O$. Similarly, $U_{\rm I} = U_1$ holds. Thus it follows:

$$\begin{align*} - U_{\rm O} = (U_{\rm I} - U_{\rm O}) \cdot \frac{R_2}{R_1 + R_2} \end{align*}$$

And from that:

$$\begin{align*} - U_O &= U_I \cdot \frac{R_2}{R_1 + R_2} - U_O \cdot \frac{R_2}{R_1 + R_2} \\ - U_O + U_O \cdot \frac{R_2}{R_1 + R_2} &= U_I \cdot \frac{R_2}{R_1 + R_2} \\ U_O \cdot (\frac{R_2}{R_1 + R_2}-1) &= U_I \cdot \frac{R_2}{R_1 + R_2} \\ \frac{U_O}{U_I} &= \frac{\frac{R_2}{R_1 + R_2}}{\frac{R_2}{R_1 + R_2}-1} \\ \frac{U_O}{U_I} &= \frac{R_2}{R_2 - (R_1 + R_2)} = \frac{R_2}{-R_1} \\ \\ \boxed{ A_V = - \frac{R_2}{R_1}} \end{align*}$$

For the second derivation, the current flow through the resistors R_1 and R_2 of the unloaded voltage divider is to be considered. These two currents I_1 and I_2 are just equal. Thus:

$$\begin{align*} I_\boxed{}=\frac{U_\boxed{}}{R_\boxed{}}=const. \quad \text{with} \: \boxed{}=\{1,2\} \end{align*}$$

respectively

$$\begin{align*} \frac{U_1}{R_1}=\frac{U_2}{R_2} \end{align*}$$

This can also be converted into a “seesaw” or mechanical analog via **similar triangles** (see Similarity_(geometry)). In the mechanical analog, the potentials are given by height.
As in the electrical case with the ground potential, a height reference plane must be chosen in the mechanical picture.
The electric currents correspond to forces (i.e., a momentum flux) - but the consideration of forces is not necessary here. [1].

Fig. 6: Inverting Amplifier - Animation

Now, if a certain height (voltage $U_\rm I$) is set, a certain height on the right side (voltage $U_\rm O$) is obtained via the force arm (resistor R_1) and load arm (resistor R_2). This is shown in figure 6 above.

In the figure, all points marked in red () can be manipulated. Accordingly, the input voltage $U_{\rm I} = U_{\rm in}$ is adjustable and automatically results in a voltage $U_{\rm O}=U_{\rm out}$. In the circuit (figure below), the resistors R_1 and R_2 can be changed.

The **input resistance of the entire circuit** $R_{\rm I}^0=\frac{U_{\rm I}}{I_{\rm I}}$ is easily obtained by considering the input side: since $\rm K1$ is at $0 ~\rm V$, $U_1 = U_\rm I$.
The complete current flowing into the input passes through resistor R_1. So, it is then true that the input resistance is $R_{\rm I} = R_1$.
At the **output resistance of the whole circuit** $R_{\rm O}^0$, there is again a parallel connection between the output resistance of the operational amplifier $R_\rm O$ and the resistor R_2.
So, the output resistance will be slightly smaller than the output resistance of the operational amplifier $R_\rm O$.

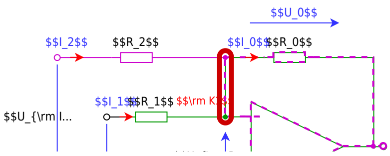
Notice: Inverting Amplifier

In the case of the inverting amplifier:

- The input voltage $U_\rm I$ is at the inverting input of the operational amplifier.
- The feedback is done by a voltage divider of R_1 and R_2.
- The voltage gain is $A_{\rm V}= - \frac{R_2}{R_2}$ and is always less than or greater than 0. However, the magnitude of the voltage gain can be greater than or less than 1.
- The input resistance of the whole circuit is defined by R_1 and is usually smaller than the input resistance of the used (real) operational amplifier. The output resistance is smaller than that of the used (real) operational amplifier.

Inverting Summing Amplifier

Fig. 7: Inverting Summing Amplifier



From the inverting amplifier another circuit can be derived, which can be seen in figure 7. Here, both the green part of the circuit and the purple part correspond to an inverting amplifier.

How can $U_\rm O$ be calculated in this circuit? To do this, it is first important to understand what is being sought. The goal is to find the relationship between output and input signals: $U_{\rm O}(U_{\rm I1}, U_{\rm I2})$. Different ways to get there, were explained in Block07 and Block08. Here we will outline a different way.

In the case of a circuit with several sources, superposition is a suitable method, in particular the superposition of the effect of all sources in the circuit. For superposition, it must be ensured that the system behaves linearly. The circuit consists of ohmic resistors and the operational amplifier. These two components give twice the output value when the input value is doubled - they behave linearly. For superposition, the effect of the two visible voltage sources $U_{\rm I1}$ and $U_{\rm I2}$ must be analyzed in the present circuit.
In **case 1** the voltage source $U_{\rm I1}$ must be considered - the voltage source $U_{\rm I2}$ must be short-circuited for this purpose. The equivalent circuit formed corresponds to an inverting amplifier across R_2 and R_0. However, there is an additional resistor R_1 between the inputs of the operational amplifier. What is the influence of this resistor? The differential voltage $U_\rm D$ between the inputs of the operational amplifier approaches 0. Thus, the following also applies to the current through R_1: $I_{1(1)} \rightarrow 0$. Thus the circuit in case 1 is exactly an inverting amplifier. For case 1, $A_{V(1)} = \frac{U_{O(1)}}{U_{\rm I1}} = - \frac{R_0}{R_1}$ and thus: $U_{O(1)}= - \frac{R_0}{R_1} \cdot U_{\rm I1}$.
Using the same procedure, **case 2** for considering the voltage source U_2 gives: $U_{\rm O(2)}= - \frac{R_0}{R_2} \cdot U_{\rm I2}$.
In superposition, the effect results from the **addition of partial effects**:

$\boxed{U_{\rm O} = \sum_i U_{\rm O(i)} = - (\frac{R_0}{R_2} \cdot U_{I2} + \frac{R_0}{R_1} \cdot U_{I1})}$.

Also, considering the node set for $\rm K1$ in figure 7 gives the same result.

The **Inverting Summing Amplifier** (also called: Summing Amplifier or Voltage Adder) can be extended to any number of inputs.
The simulation above shows the superposition of several inputs. Depending on the resistances at the different inputs, a different current flows into the circuit.

This circuit was used in analog audio mixers. This allows a combination of several signals with different gains (by the input resistors R_i with $i=1, ..., n$). Furthermore, the overall gain can be changed by R_0. A big advantage of this circuit is also that the summation at node $\rm K1$ is done on potential $U_\rm D$. This means that capacitive interference concerning the ground potential (and therefore the case) is virtually non-existent.

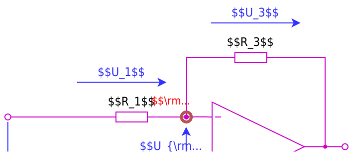
A very similar concept allows the construction of a Digital-Analog Converter, DAC.

Differential Amplifier / Subtractor

In addition to the (reverse) adder, there is also a circuit for subtracting two input values. This circuit became the core of the introductory example. But also in the simulation below this circuit is shown in another example: In this case, a differential input signal is shown on the left. Differential means that the signal on one line is , not transmitted concerning a reference voltage (usually ground potential) on a second line. Instead, the signal is transmitted to both lines in opposite directions. If a disturbance acts equally on both lines (which is often the case when lines are close to each other), the effect of the disturbance can be eliminated by forming the difference.

How can the relationship $U_{\rm O}(U_{\rm I1}, U_{\rm I2})$ between output and input signals be determined for this circuit?

Fig. 8: Differential Amplifier



Again, various network analysis concepts could be used to look at the circuit (e.g. superposition or mesh and node sets). Again, another possibility is to split the circuit as color-coded in the figure 8.
The green part shows a voltage divider $R2 + R4$. Since the input resistance of the operational amplifier is very large, this voltage divider is unloaded. The voltage at node $\rm K2$ or at the noninverting input $U_\rm p$ is just given by the voltage divider: $U_{\rm p} = U_{\rm I2}\cdot \frac{R_4}{R_2 + R_4}$.
The violet part corresponds to an inverting amplifier, but the voltage at the node $\rm K1$ or at the inverting input $U_\rm m$ is just equal to $U_\rm p$ due to the feedback, since $U_\rm D \rightarrow \infty$. Thus, the current flowing into node $\rm K1$ via R_1 results from $I_1=\frac{U_{\rm I1} - U_\rm p}{R_1}$. The output voltage is given by $U_{\rm O} = U_{\rm p} - U_3$, where the voltage U_3 is given by the resistance R_3 and the current through R_3. The current through R_3 is just the same as the current through R_1, i.e. I_1.

The result is:
$ U_{\rm O} = U_{\rm I2}\cdot \frac{R_4}{R_2 + R_4} - R_3 \cdot \frac{U_{\rm I1} - U_{\rm p}}{R_1} $
$ U_{\rm O}= U_{\rm I2}\cdot \frac{R_4}{R_2 + R_4} - U_{\rm I1} \cdot \frac{R_3}{R_1} + U_{\rm I2} \cdot (\frac{R_3}{R_1}\cdot \frac{R_4}{R_2 + R_4})$
$\boxed{U_{\rm O}= U_{\rm I2}\cdot \frac{R_4}{R_2 + R_4} \frac{R_1 + R_3}{R_1} - U_{\rm I1} \cdot \frac{R_3}{R_1}}$

Fig. 9: Differential Amplifier - Animation

Please click to see the animation!

Two simplifications should be considered here:

1. If $R_1 = R_2$ and $R_3 = R_4$ are chosen, the equation further simplifies to:
   $\boxed{U_{\rm O} = \frac{R_3}{R_1}\cdot(U_{\rm I2}-U_{\rm I1})}$.
   This variant can be found in various measurement circuits.

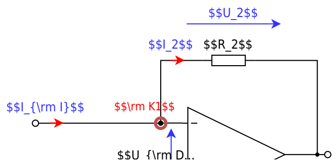
2. Alternatively, if $R_1 = R_3$ and $R_2 = R_4$ is chosen, the result is:
   $\boxed{U_{\rm O}= U_{\rm I2}-U_{\rm I1}}$
   This would also result in case 1. if $R_1 = R_2 = R_3 = R_4$ is chosen.

The animation shows how the 2nd case would result in similar triangles. The connection of the two “seesaws” at the point $\rm K_1 K_2$ is caused by the operational amplifier, through which the voltage $U_\rm p$ and $U_\rm m$ converge to $U_\rm D \rightarrow 0$.

A big advantage of this circuit is that even very large voltages can be used as input voltage, if $R_1 \gg R_3$ and $R_2 \gg R_4$ are chosen. This would divide the input voltages down and display a fraction of the difference as the result. The main drawback of the circuit is that the gain/attenuation depends on more than one resistor. This makes a quick choice of gain difficult.

Current-Voltage-Converter

Fig. 1: Current-Voltage-Converter



In figure 1 one can see the circuit of a current-voltage converter. The current-to-voltage converter changes its output voltage based on an input current. This circuit is also called a transimpedance amplifier because here the transfer resistance - that is, the trans-impedance - represents the gain.

Generally, the gain was expressed as $$A={ {\rm output} \over {\rm input} }$$.

In the case of the current-to-voltage converter, the gain is defined as:

$$R = {{U_{\rm out}} \over I_{\rm in}} ={{U_{\rm o}} \over I_{\rm I}} = - R_1$$

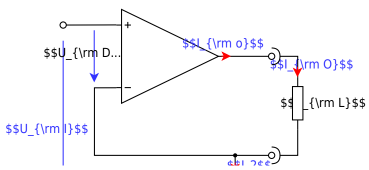
R_1 is the resistor used in the circuit.

In the simulation, the slider on the right ("Current of current source") can be varied. This changes the input current and thus the output voltage.

This circuit can be used, for example, to read a photodiode in volt-free circuit (further explanation and integrated circuit tsl250r.pdf).

Voltage-to-Current Converter

Fig. 2: Voltage-to-Current Converter



Next, consider the voltage-to-current converter. With this, an output current is set proportional to an input voltage.

Here, the general gain $$A={ {\rm output} \over {\rm input} }$$ to

$$S = {{I_{\rm out}} \over U_{\rm in}} = {{I_{\rm o}} \over U_{\rm I}}$$

The quantity S is called the transfer conductance.

This circuit can be used, for example, to generate a voltage-regulated current source.
In practical applications, often specialized amplifiers, called Operational Transconductance Amplifier (transconductance from transmission conductance), are used.

Applications

Programmable Gain Amplifier

Often in applications an analog signal is too small to process (e.g. to digitalize it afterward).
To amplify it an OpAmp can be used. However, for a wide input range, it might be beneficial to have an adjustable scale.

This can be done with a simple non-inverting amplifier combined with a resistor network as seen in the next simulation.
In this case, a so-called **single-ended** input is used. This means the input voltage is always referred to the ground.

When the signal is not referred to the ground, the following circuit based on an instrumentation amplifier can be used.
In this case, the input signal is **differential**. Referred to the ground the input signal (here the difference of $5 ~\rm mV$) can have an offset voltage with regard to the ground.
An example of this setup is the INA 351.

Common pitfalls

- ...

Exercises

Worked examples

Exercise 3.5.1 inverting amplifier

1. Derive the voltage gain $A_{\rm V}= {{U_{\rm O}}\over{U_{\rm I}}}$ for the inverting amplifier.
Use the procedure that was used for the non-inverting amplifier.

- What is required?
- Number of variables?
- Number of necessary equations?
- Set up the known equations
- Derivation of the voltage gain

Take into account that for the differential gain $A_\rm D$ of the ideal OPV applies: $A_\rm D \rightarrow \infty$. And the following also applies: $1/A_\rm D \rightarrow 0$
But the following doesn't always apply: ${{C}\over{U_x \cdot A_\rm D}} \rightarrow 0$, for an unknown constant C and a voltage U_x!

Solution for “What is required?”

$A_{\rm V} = \frac{U_{\rm O}}{U_{\rm I}}$

Solution for “Number of variables?”

- 5 voltages: $U_{\rm I}$, $U_{\rm 1}$, $U_{\rm D}$, U_2, $U_{\rm O}$
- 5 currents: I_1, $I_{\rm m}$, $I_{\rm p}$, I_2, $I_{\rm o}$
- --> 10 variables

Solution for “Number of necessary equations?”

9, since one equation is to be determined

Solution for “Set up the known equations”

- Fundamental equation: (1) $U_{\rm O} = A_{\rm D} \cdot U_{\rm D}$
- Golden rules:
  - $R_{\rm D} \rightarrow \infty$, and thus (2) + (3) $I_{\rm m} = I_{\rm p} = 0$
  - $R_{\rm O} = 0$
  - (4) $A_{\rm D} \rightarrow \infty$ and with (1) $U_{\rm D} = \frac{U_{\rm O}}{A_{\rm D}} \rightarrow 0$
- Mesh equations
  - Mesh 1: (5) $-U_{\rm I} + U_1 - U_{\rm D} = 0$
  - Mesh 2: (6) $U_{\rm D} + U_2 + U_{\rm O} = 0$
- Node equation: (7) $I_1 - I_2 + 0 = 0$
- Resistors:
  - (8) $R_1 = \frac{U_1}{I_1}$
  - (9) $R_2 = \frac{U_2}{I_2}$

Solution for “Derivation of the voltage gain”

\begin{align*} A_V &= \frac{U_{\rm O}}{U_{\rm I}} \quad | \quad \text{using (5) and (6)} \\

A_V &= \frac{- U_{\rm D} - U_2}{U_1 - U_{\rm D}} \quad | \quad \text{using (8) and (9)} \\ A_V &= \frac{- U_{\rm D} - R_2 \cdot I_2}{R_1 \cdot I_1 - U_{\rm D}} \quad | \quad \text{using (1) }\\ A_V &= \frac{- R_2 \cdot I_2}{R_1 \cdot I_1} \quad | \quad \text{using (7)} \\ A_V &= -\frac{R_2 }{R_1 } \end{align*}

2. Which type of amplifier circuit (inverting or non-inverting amplifier) has the lower input resistance? Why?

Solution“

The input resistance of the **inverting amplifier** is the resistor R_1.
The input resistance of the **non-inverting amplifier** is **larger than the input resistance of the op-amp**.
Therefore, the inverting amplifier has the lower input resistance.

Exercise 3.5.2. Variations of the non-inverting amplifier

Below you will find circuits with an ideal operational amplifier, which are similar to the non-inverting amplifier and whose voltage gain A_V must be determined.

<u>Assumptions</u>

- $R_1 = R_3 = R_4 = R$
- $R_2 = 2 \cdot R$
- $U_\rm I$ comes from a low-resistance source
- $U_\rm O$ is due to a high-resistance consumer

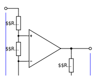
<u>Exercises</u>

1. Enter the voltage gain $A_\rm V$ for each circuit. A detailed calculation as before is not necessary.
2. For Figure 7, indicate how the voltage gain can be determined.
3. Generalize with the following justifications:
    1. How has a short circuit of the two OPV inputs must be taken into account?
    2. How do resistances have to be considered in the following cases:
        1. with one terminal (so “one connector”) directly and exclusively on an OPV input,
        2. with both terminals each directly connected to an OPV input.
4. In which circuits do resistors R_3 and R_4 represent an unloaded voltage divider?

To approach the problems, you should try to use the knowledge from the inverting amplifier. It can be useful to simulate the circuits via Falstad-Circuit or Tina TI. In the first two circuits, tips can be seen under the illustration as support.

<u>Important</u>: As always in your studies, you should try to generalize the knowledge gained from the task.

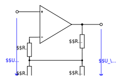
Abb. 1



Hints

- How high is the current flow into the inverting and non-inverting input of an ideal operational amplifier? What voltage drop would there be across a resistor whose one connection only leads to one input of the operational amplifier? (R_3)?
- The operational amplifier always tries to output enough current at the output so that the required minimum voltage is between the inverting and non-inverting input $U_\rm D$ results. How big can $U_\rm D$ be accepted? Can this voltage also via a resistor (R_4) being constructed?

Abb. 2



Hints

- How much current must flow through $R_4 = R$ so that the expected voltage U_4 results?
- How much current must flow through $R_2 = 2 \cdot R$ fließen?
- How much current must flow through $R_1 = R$? How high is the voltage at R_1?

Solution for (1) + (2)

Fig. 1

- Between the inverting and non-inverting input, only $U_D \rightarrow 0$ is present. As long as $R_4 > 0$, this small voltage can exist there. Therefore, R_4 can be replaced by an open circuit.
- Since no current flows through R_3, there is no voltage difference across R_3. Thus, R_3 can be replaced by a short circuit.
- This results in a non-inverting amplifier with $A_V = 3/2$

Fig. 2

- No current flows through R_3, since no current can flow into the op-amp. Therefore, there is no voltage difference across R_3, and R_3 can be replaced by a short circuit.
- R_4 and R_2 are in parallel and yield $R_g = \frac{2}{3} R_1$
- The gain thus becomes $A_V = \frac{R_g + R_1}{R_g} = 2.5$

Abb. 3

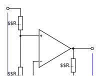
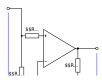


Abb. 4



Solution for (3) + (4)

Fig. 3

- R_3 and R_4 form an unloaded voltage divider.
- Thus, the input voltage is halved.
- This results in a non-inverting amplifier with $A_V = \frac{R_2 + R_1}{R_2} = 0.75$

Fig. 4

- No current flows through R_3, since no current can flow into the op-amp. Therefore, there is no voltage difference across R_3, and R_3 can be replaced by a short circuit.
- Since the input voltage comes from a low-impedance voltage source, R_4 is negligible compared to the internal resistance of the source. Therefore, R_4 can be replaced by an open circuit.
- The overall gain thus becomes $A_V = 3/2$

Abb. 5

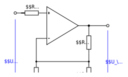
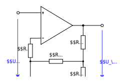


Abb. 6



Solution for (5) + (6)

Fig. 5

- No current flows through R_3, since no current can flow into the op-amp. Therefore, there is no voltage difference across R_3, and R_3 can be replaced by a short circuit.
- R_4 and R_2 are in parallel and yield $R_g = \frac{2}{3} R_1$
- The gain thus becomes $A_V = \frac{R_g + R_1}{R_g} = 2.5$

Fig. 6

- No current flows through R_4, since no current can flow into the op-amp. Therefore, there is no voltage difference across R_4, and R_4 can be replaced by a short circuit.
- Since the inverting input is thus at 0 V, the output voltage becomes the maximum possible output voltage of the op-amp.

Abb. 7

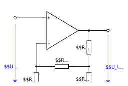
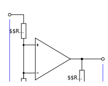


Abb. 8



Solution for (7) + (8)

Fig. 7

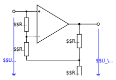
- The input voltage is applied across R_4, so a current $I = U_E / R$ flows through it.

This current also flows through R_3. Therefore, the voltage across both is $2 \cdot U_E$.
- The resistors R_2, R_3, and R_4 form $R_g = (R_3 + R_4)||R_2 = 2R||2R = R$
- Thus $R_g = R_1$, and the output voltage U_A is twice the voltage across R_g, so the overall gain is $A_V = 4$

Fig. 8

- The inverting and non-inverting inputs are shorted, so the output voltage is 0.

Abb. 9



Solution for (9)

Fig. 9

- Between the inverting and non-inverting input, only \$U_D \rightarrow 0\$ is present. As long as \$R_3 > 0\$, this small voltage can exist there. Therefore, \$R_3\$ can be replaced by an open circuit.
- Since no current flows through \$R_4\$, there is no voltage difference across \$R_4\$. Thus, \$R_4\$ can be replaced by a short circuit.
- This results in a non-inverting amplifier with \$A_V = \frac{R_2 + R_1}{R_2} = 1.5\$

Exercise 3.5.4. Conversion of a unipolar signal into a bipolar signal

You work in the company “HHN Mechatronics & Robotics” and are supposed to generate a bipolar signal ($-10 ~\rm V ... + 10 ~\rm V$) from a unipolar signal of a digital-to-analog converter ($0 ... 5 ~\rm V$) in a project. A colleague recommended the circuit shown on the right.

1. First, analyze what change is made by pressing the switch S. How does the output signal change?
2. Try to determine mathematically the relationship of $U_\rm O$ and $U_\rm I$ as $U_{\rm O}(U_{\rm I})$ by superposition.
3. The circuit still has the problem that for a positive positive half-wave the output is still negative. Which additional circuit must be provided so that this problem can be solved?

Exercise 3.5.5. Linear Voltage Regulator

In order to get a constant (lower) voltage from a higher voltage input or a source with a broader spread of the voltage (e.g. a battery) often linear regulators are used. One example could be to get $5 ~\rm V$ from the car battery voltage (between $11 ~\rm V...14 ~\rm V$) for a microcontroller in a control unit e.g. the brake control unit. Linear regulator here means that a transistor as a variable resistor is used to drop the unwanted voltage.

Below, two types of such linear regulators are shown

1. The first simulation shows a simple series regulator with a FET. "Series" here marks the fact that the transistor is in series to the load resistor $R_\rm L$. The Zener diode D has a current limiting series resistors $R_\rm D$ ahead. By the voltage divider of $R_\rm D$ and D, a relatively constant voltage will be created.
2. The second simulation shows a more sophisticated circuit. Here, there is feedback from the output of the transistor back to the transistor controlling voltage. This feedback is given by $R1$, $R2$, and the operational amplifier.

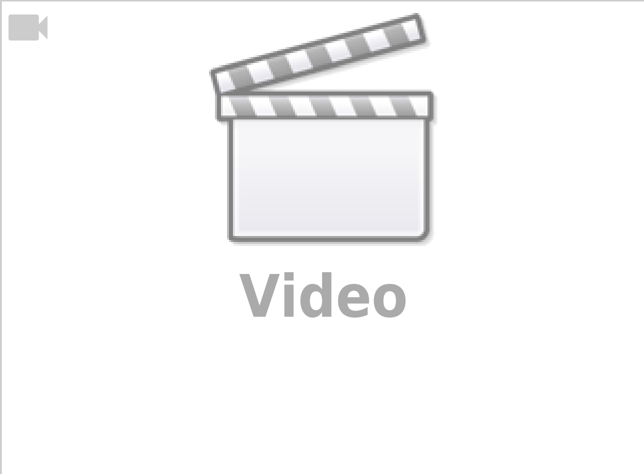
Tasks

- In both simulations there are two sliders on the right-hand side:
  - *Input Voltage*, which changes the ingoing voltage between $5~\rm V...20~\rm V$
  - *Load Resistance*, which changes the load on the output between $10~\Omega...1~\rm k\Omega$
  Play with these sliders and look for the differences! What are these?
- The lower simulation with the operational amplifier is also called "**L**ow **D**rop**O**out" (**LDO**). The dropout is the minimum voltage difference on the transistor. How can the terminology low dropoff can be explained?
- To which primitive OpAmp circuit does the LDO circuit (R_1, R_2 and OpAmp) look similar to?
  How can the controlling of the transistor input voltage $U_{\rm GS}$ be explained?
- Given a load resistor of $R_\rm L=1~\rm k\Omega$, an input voltage $U_\rm I=20~\rm V$, and an output voltage $U_\rm O=5~\rm V$, what is the dissipated power on the load and on the transistor?
- One LDO is the TPS746.
  - What is the Pin $\rm FB$ for?
  - How does the LM7805 differ regarding the set-up in a circuit?

Embedded resources

Non-inverting operation amplifier circuit

[1] To complete the mechanical analogue of the setup, one can assume that there is an external "force source". This always acts in such a way that it



always lands on the height reference surface at the point corresponding to the virtual mass