

Exam Winter Semester 2022

Student Group

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Table of Contents

- Exam Winter Semester 2022** 3
- Additional permitted Aids 3
- Hits 3
- Tasks 3
- Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) 3
- Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) 3
- Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) 4
- Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) 4
- Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) 5
- Exercise E2 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) 7
- Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) 9
- Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) 13
- Exercise E4 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) 17
- Exercise E1 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) 18
- Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) 20
- Exercise E4 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) 20
- Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) 20

written test, WS2022)	21
Exercise E1 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)	21
Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)	22
Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)	25

Exam Winter Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected. The power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary. Calculate the current I needed to operate it.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ I &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected.

Result power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it for heating elements.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = 10^{-3} \text{ } \Omega$ (Wrong calculation)

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator explains a temperature sensitive component. The thermistor has a resistance of $10 \text{ k} \Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$

Result
 The temperature inside the refrigeration system can reach down to -40°C .
 Calculate the resistance of the thermistor at -40°C .
 Resistance of the resistor R depends on the current I and generated heat. Therefore, a solution is to use the heat flow up the refrigeration system.
 Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad || \quad R = 10 \text{ k} \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2\right)$$

Exercise E1 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A resistor exhibits a temperature coefficient of resistance of $\alpha = 0.01 \text{ K}^{-1}$ and a nominal resistance of $R_0 = 10 \text{ k}\Omega$ at $T_0 = 25^\circ\text{C}$. Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 (1 + \alpha \Delta T + \beta \Delta T^2) = 10 \text{ k}\Omega (1 + 0.01(-40 - 25) + 71 \cdot 10^{-6}(-40 - 25)^2) = 6.5 \text{ k}\Omega$$

Resistor transfer resistor R in parallel with the circuit and generate heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 (1 + \alpha \Delta T + \beta \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

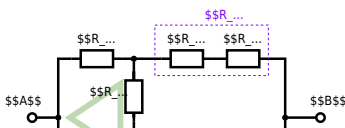
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall hold: $R_1 = 200 \text{ }\Omega$, $R_2 = R_3 = 100 \text{ }\Omega$, $R_4 = 10 \text{ }\Omega$ and the voltage $U = 10 \text{ V}$.
Result: $R_B = 132.8 \text{ }\Omega$.

Solution

$$R_{\text{B}} = 132.8 \text{ }\Omega$$

Now a wye-delta transformation is necessary.

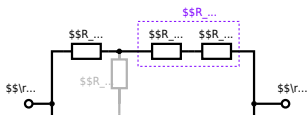


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{\{500 \sim\Omega \cdot 200 \sim\Omega\}}{500 \sim\Omega + 200 \sim\Omega}$$

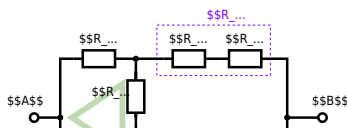
Exercise E2 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the voltage $U = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

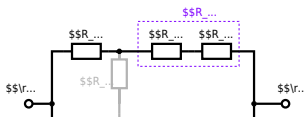


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



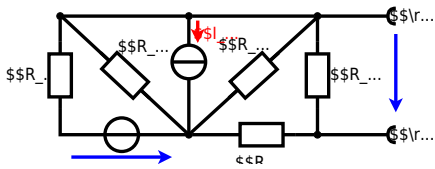
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{eq}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega}$$

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

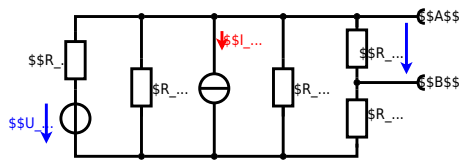
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \, \text{V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$



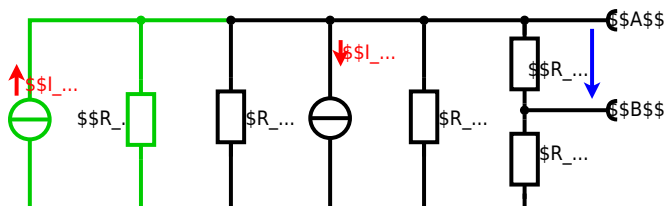
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_s=6.0 \text{ V}$, $R_2=10 \Omega$, $I_4=4.2 \text{ A}$, $R_3=10 \Omega$, $R_4=7.5 \Omega$, $R_5=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_4 + I_{24} \cdot R_{67}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

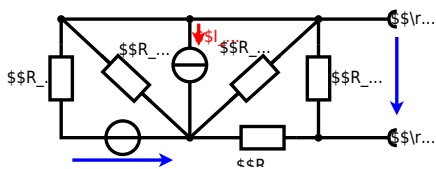
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

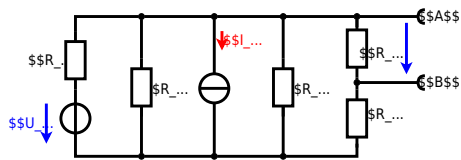
$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



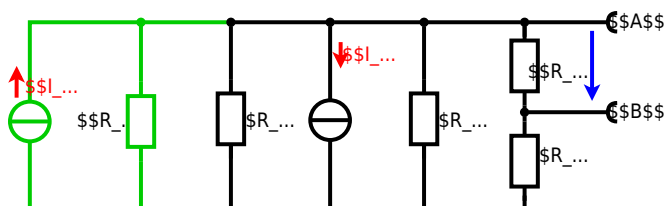
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{1}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

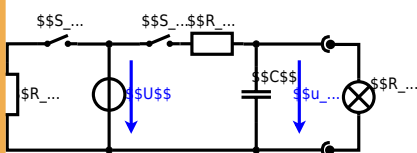
Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a RC circuit consisting of a DC voltage source U , a resistor R_1 , a resistor R_2 , and a capacitor C . The switch S_1 is initially open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

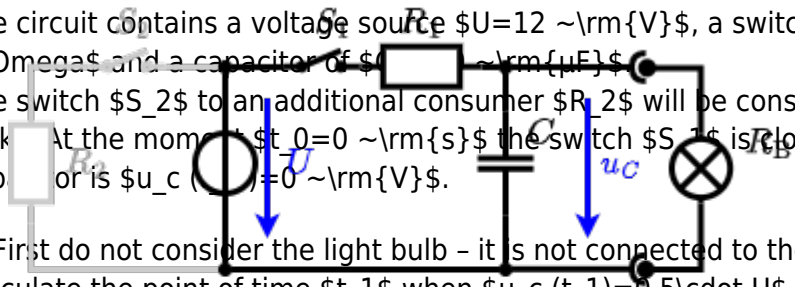
Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U_{eq} is given by $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$ and the internal resistance $R_{eq} = R_1 || R_2$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

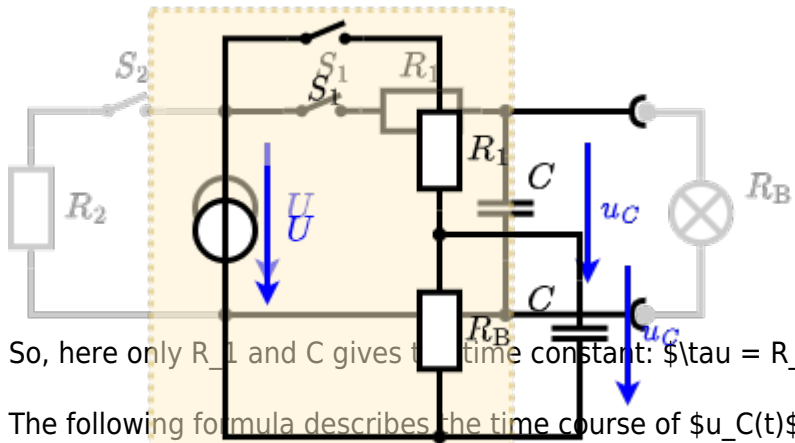


The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$. An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E1 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a 12 V DC voltage source, a $20\text{ }\Omega$ resistor, a $100\text{ }\mu\text{F}$ capacitor, and a $20\text{ }\Omega$ resistor. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

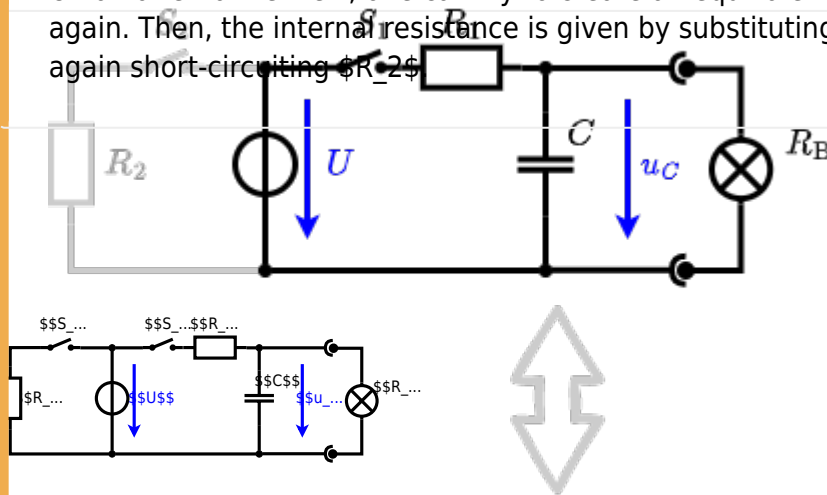
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

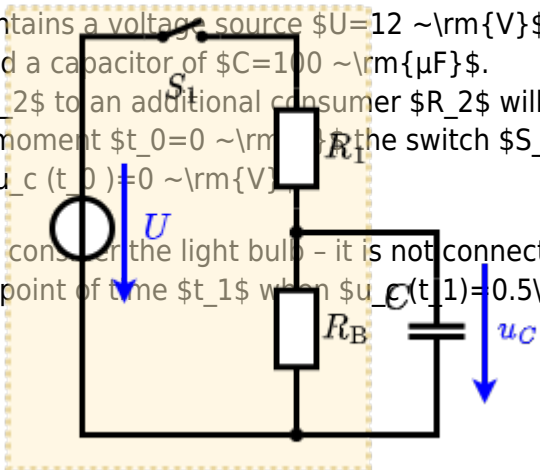
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$: $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the admittance $\underline{Y} = 0.24 - j0.2$ S, determine the current \underline{I} through the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned complex impedance can be extracted and split (align) in phase (align) late left (align) $+5 \{ \text{rm j} \}$ \Omega end{align*}

Solution
.. Calculation of physical values of the two components.
Solution $\begin{aligned} R &= \frac{1}{\text{Re}\{Y\}} = \frac{1}{0.24} = 4.17 \text{ } \Omega \\ X_L &= \frac{1}{\text{Im}\{Y\}} = \frac{1}{-0.2} = -5 \text{ } \Omega \end{aligned}$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{\frac{1}{0.24 - j0.2}} = 50 (0.24 + j0.2) = 12 + j10 \text{ A}$$

The current \underline{I} has a magnitude of 15.62 A and a phase angle of $\varphi_i = \arctan\left(\frac{10}{12}\right) = 39.8^\circ$.
The voltage across the resistor is $\underline{U}_R = \underline{I} R = (12 + j10) \cdot 4.17 = 50 \angle 0^\circ$ V.
The voltage across the inductor is $\underline{U}_L = \underline{I} X_L = (12 + j10) \cdot (-j5) = 50 \angle 90^\circ$ V.
The phase shift between \underline{U} and \underline{I} is $\varphi = 39.8^\circ$.
The phase shift between \underline{U}_R and \underline{I} is $\varphi_R = 0^\circ$.
The phase shift between \underline{U}_L and \underline{I} is $\varphi_L = 90^\circ$.

Exercise E4 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the admittance $\underline{Y} = 0.24 - j0.2$ S, determine the current \underline{I} through the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned complex impedance can be extracted and split (align) in phase (align) late left (align) $+5 \{ \text{rm j} \}$ \Omega end{align*}

Solution
.. Calculation of physical values of the two components.
Solution $\begin{aligned} R &= \frac{1}{\text{Re}\{Y\}} = \frac{1}{0.24} = 4.17 \text{ } \Omega \\ X_L &= \frac{1}{\text{Im}\{Y\}} = \frac{1}{-0.2} = -5 \text{ } \Omega \end{aligned}$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{\frac{1}{0.24 - j0.2}} = 50 (0.24 + j0.2) = 12 + j10 \text{ A}$$

The current \underline{I} has a magnitude of 15.62 A and a phase angle of $\varphi_i = \arctan\left(\frac{10}{12}\right) = 39.8^\circ$.
The voltage across the resistor is $\underline{U}_R = \underline{I} R = (12 + j10) \cdot 4.17 = 50 \angle 0^\circ$ V.
The voltage across the inductor is $\underline{U}_L = \underline{I} X_L = (12 + j10) \cdot (-j5) = 50 \angle 90^\circ$ V.
The phase shift between \underline{U} and \underline{I} is $\varphi = 39.8^\circ$.
The phase shift between \underline{U}_R and \underline{I} is $\varphi_R = 0^\circ$.
The phase shift between \underline{U}_L and \underline{I} is $\varphi_L = 90^\circ$.

The absolute value of the impedance is $\sqrt{(0.24 \sim \Omega)^2 + (4.68 \sim \Omega)^2} = 4.7 \sim \Omega$
 The phase $\varphi = \arctan\left(\frac{4.68 \sim \Omega}{0.24 \sim \Omega}\right) = 86.8^\circ$
 With the complex part comes the physical value: $X_L = \omega L = 2\pi \cdot 4 \cdot 10^6 \cdot 1.00 \cdot 10^{-6} = 25.1 \sim \Omega$
 The phase $\varphi = \arctan\left(\frac{25.1 \sim \Omega}{10.0 \sim \Omega}\right) = 68.3^\circ$
 The phase difference $\Delta\varphi = 86.8^\circ - 68.3^\circ = 18.5^\circ$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

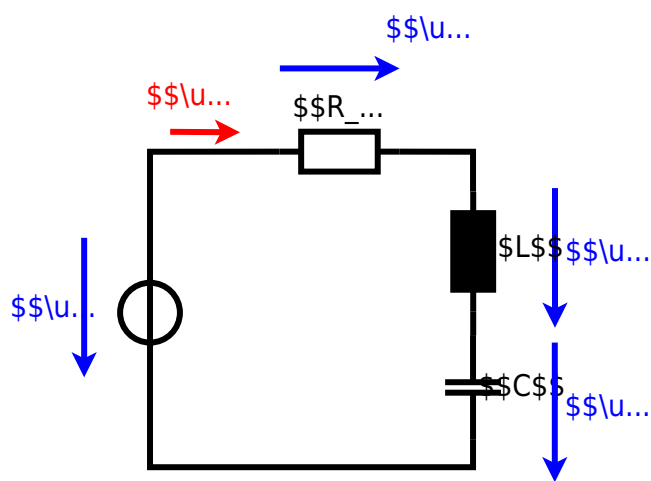
At a series circuit with a resistor $R_1 = 1.00 \sim \Omega$, a capacitor $C_1 = 40 \sim \text{nF}$ and an inductor $L_1 = 4.7 \sim \mu\text{H}$ in AC with a voltage $U = 10 \sim \text{V}$ and a frequency $f = 450 \sim \text{kHz}$.
 Result: $R_1 = 1.00 \sim \Omega$, $X_C = \frac{1}{2\pi \cdot 450 \cdot 10^3 \cdot 40 \cdot 10^{-9}} = 0.89 \sim \Omega$, $X_L = 2\pi \cdot 450 \cdot 10^3 \cdot 4.7 \cdot 10^{-6} = 1.33 \sim \Omega$
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \sim \text{nF}$ at $f_1 = 4 \sim \text{MHz}$.

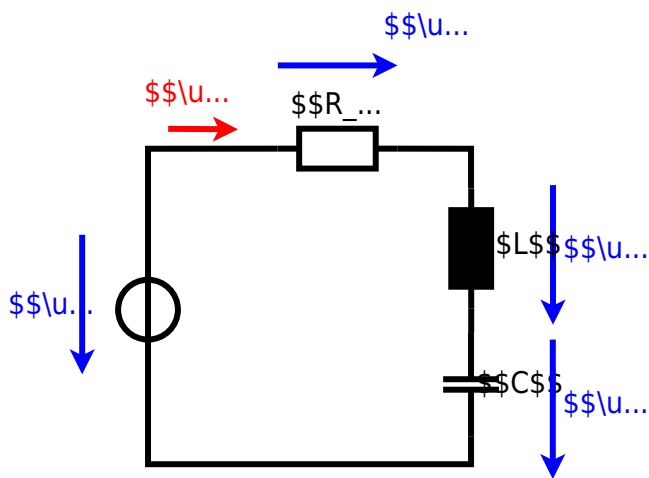
Solution
 Solution $R_1 = 1.00 \sim \Omega$
 Solution $R_2 = 10.0 \sim \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_2 and C_2 .
 The equivalent impedance for R_2 and C_2 combined is given by $Z = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$
 Since Z is perpendicular to R_2 , this can be simplified to $Z = \frac{R_2 \cdot X_C}{X_C}$
 $Z = R_2$ is perpendicular to X_C (It has to, since R_2 is perpendicular to X_C)
 Therefore, the resulting current of the parallel circuit is given as:
 $I = \frac{U}{\sqrt{R_2^2 + X_C^2}}$
 This can be rearranged to get $R_2 = \frac{U}{I} \cdot \sqrt{1 - \left(\frac{X_C}{U/I}\right)^2}$
 Back to the first formula: $R_2 \cdot X_C = X_C \cdot \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$
 $R_2 = \frac{X_C \cdot \sqrt{R_2^2 + X_C^2}}{X_C}$

Exercise E1 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

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