Block 04 — Complex Calculus in EE

Student Group

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Learning objectives

After this 90-minute block, you

- know how sine variables can be symbolized by a vector.
- know which parameters can determine a sinusoidal quantity.
- graphically derive a pointer diagram for several existing sine variables.
- can plot the phase shift on the vector and time plots.
- can add sinusoidal quantities in vector and time representation.
- know and apply the impedance of components.
- know the frequency dependence of the impedance of the components. In particular, you should know the effect of the ideal components at very high and very low frequencies and be able to apply it for plausibility checks.
- are able to draw and read pointer diagrams.
- know and apply the complex value formulas of impedance, reactance, and resistance.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Representation and Interpretation

Up to now, we used for the AC signals the formula $x(t)= \sqrt{2} X \cdot \sin (\omega t + \varphi_x)$ - which was quite obvious.

However, there is an alternative way to look at the alternating sinusoidal signals. For this, we look first at a different, but already a familiar problem (see figure 1).

1. A mechanical, linear spring with the characteristic constant D is displaced due to a mass m in the Earth's gravitational field. The deflection only based on the gravitational field is X_0.
2. At the time $t_0=0$, we deflect this spring a bit more to $X_0 + x(t_0)=X_0 + \hat{X}$ and therefore induce energy into the system.
3. When the mass is released, the mass will spring up and down for $t>0$. The signal can be shown as a shadow when the mass is illuminated sideways.
   For $t>0$, the energy is continuously shifted between potential energy (deflection $x(t)$ around X_0) and kinetic energy (${{\rm d}\over{{\rm d}t}}x(t)$)
4. When looking onto the course of time of $x(t)$, the signal will behave as: $x(t)= \hat{X} \cdot \sin (\omega t + \varphi_x)$
5. The movement of the shadow can also be created by the sideways shadow of a stick on a rotating disc.
   This means, that a two-dimensional rotation is reduced down to a single dimension.

Fig. 1: interpretation of sinusoidal deflection of a spring

The transformation of the two-dimensional rotation to a one-dimensional sinusoidal signal is also shown in figure 2.

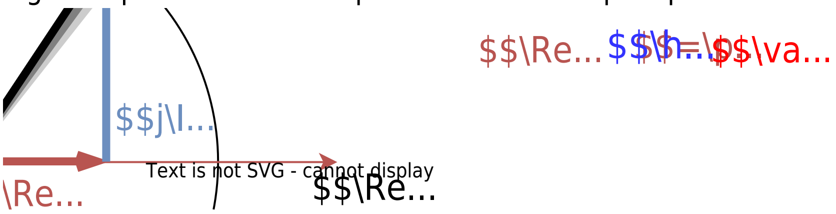
Fig. 2: Creation of the sinusoidal signal from a rotational movement

Click on the box “animate?”
press here for the animation

The two-dimensional rotation can be represented with a complex number in Euler's formula. It combines the exponential representation with real part $\Re$ and imaginary part $\Im$ of a complex value: $$ \underline{x}(t)=\hat{X}\cdot {\rm e}^{{\rm j}(\omega t + \varphi_x)} = \Re(\underline{x}) + {\rm j}\cdot \Im(\underline{x})$$

For the imaginary unit ${\rm i}$ the letter ${\rm j}$ is used in electrical engineering since the letter ${\rm i}$ is already taken for currents.

Fig. 3: representation of a phasor on the complex plane



ate System

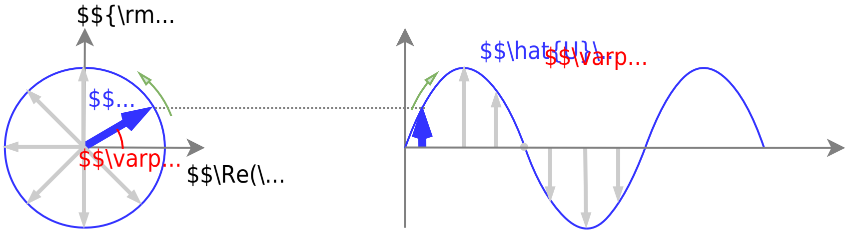
Complex Current and Voltage

The concepts of complex numbers shall now be applied to voltages and currents. Up to now, we used the following formula to represent alternating voltages:

$$u(t)= \sqrt{2} U \cdot \sin (\varphi)$$

This is now interpreted as the instantaneous value of a complex vector $\underline{u}(t)$, which rotates given by the time-dependent angle $\varphi = \omega t + \varphi_u$.

Fig. 4: representation of a voltage phasor on the complex plane



The parts on the complex plane are then given by:

1. The real part $\Re{(\underline{u}(t))} = \sqrt{2}U \cdot \cos (\omega t + \varphi_u)$
2. The imaginary part $\Im{(\underline{u}(t))} = \sqrt{2}U \cdot \sin (\omega t + \varphi_u)$

This is equivalent to the complex phasor $\underline{u}(t)=\sqrt{2}U \cdot {\rm e} ^{{\rm j} (\omega t + \varphi_u)}$

The complex phasor can be separated: \begin{align*} \underline{u}(t) &=\sqrt{2} U \cdot {\rm e}^{{\rm j} (\omega t + \varphi_u)} \\ &=\sqrt{2}\color{blue} {U \cdot {\rm e}^{{\rm j} \varphi_u}} \cdot {\rm e}^{{\rm j} \omega t} \\ &=\sqrt{2}\color{blue}{\underline{U}} \cdot {\rm e}^{{\rm j} \omega t} \\ \end{align*}

The **fixed phasor** (in German: *komplexer Festzeiger*) of the voltage is given by $\color{blue}{\underline{U}}= \color{blue}{U \cdot e ^{j \varphi_u}} $

Generally, from now on not only the voltage will be considered as a phasor, but also the current $\underline{I}$ and derived quantities like the impedance $\underline{X}$.
Therefore, the known properties of complex numbers from Mathematics 101 can be applied:

- A multiplication with j equals a phase shift of $+90°$
- A multiplication with ${{1}\over{j}}$ equals a phase shift of $-90°$

Complex Impedance

Introduction to Complex Impedance

The complex impedance is “nearly” similar calculated like the resistance. In the subchapters before, that impedance Z was calculated by $Z=\frac{U}{I}$.
Now the complex impedance is:

\begin{align*} \underline{Z}&=\frac{\underline{U}}{\underline{I}} \\ &= \Re{(\underline{Z})} + {\rm j} \cdot \Im{(\underline{Z})} \\ &= R + {\rm j} \cdot X \\ &= Z \cdot {\rm e}^{{\rm j} \varphi} \\ &= Z \cdot (\cos \varphi + j \cdot \sin \varphi) \end{align*}

With

- the resistance R (in German: *Widerstand*) as the pure real part
- the reactance X (in German: *Blindwiderstand*) as the pure imaginary part
- the impedance Z (in German: *Scheinwiderstand*) as the complex number given by the complex addition of resistance and the reactance as a complex number

The impedance can be transformed from Cartesian to polar coordinates by:

- $Z=\sqrt{R^2 + X^2}$
- $\varphi = \arctan \frac{X}{R} $

The other way around it is possible to transform by:

- $R = Z \cos \varphi$
- $X = Z \sin \varphi$

Application on pure Loads

With the complex impedance in mind, the table ## can be expanded to:

Load 		**integral representation **	**complex impedance $\underline{Z}={{\underline{U}}\over{\underline{I}}}$**	**impedance $Z $**	**phase $\varphi $**
Resistance	R	${u} = R \cdot {i}$	$Z_R = R $	$Z_R = R $	$\varphi_R = 0$
Capacitance	C	${u} ={{1}\over{C}}\cdot \int {\rm i} dt$	$Z_C = {{1}\over{{\rm j}\omega \cdot C}} = {{-{\rm j}}\over{\omega \cdot C}}$	$Z_C = {{1}\over{\omega \cdot C}}$	$\varphi_C = -{{1}\over{2}}\pi \hat{=} -90°$
Inductance	L	${u} = L \cdot {{\rm d}\over{{\rm d}t}} {i}$	$Z_L = {\rm j} \omega \cdot L $	$Z_L = \omega \cdot L $	$\varphi_L = +{{1}\over{2}}\pi \hat{=} +90°$

Tab. 1: Formulas for the different pure loads

The relationship between ${\rm j}$ and integral calculus should be clear:

1. The derivative of a sinusoidal value - and therefore a phasor - can simply be written as “$\cdot

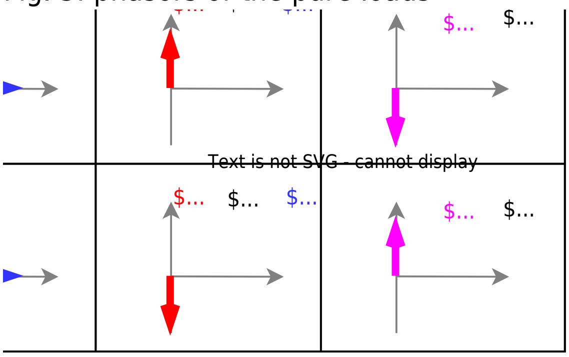
{\rm j}\omega$”,
which also means a phase shift of $+90°$:
\begin{align*}{{\rm d}\over{{\rm d}t}} {\rm e}^{{\rm j}(\omega t + \varphi_x)} = {\rm j} \cdot {\rm e}^{{\rm j}(\omega t + \varphi_x)}\end{align*}

2. The integral of a sinusoidal value - and therefore a phasor - can simply be written as “$\cdot (-{{1}\over{ {\rm j}\omega}})$”,
which also means a phase shift of $-90°$.[1]

\begin{align*} \int {\rm e}^{{\rm j}(\omega t + \varphi_x)} = {{1}\over{\rm j\omega}} \cdot {\rm e}^{{\rm j}(\omega t + \varphi_x)} = -{{\rm j}\over{\omega}} \cdot {\rm e}^{{\rm j}(\omega t + \varphi_x)} \end{align*}

Once a fixed input voltage is given, the voltage phasor $\underline{U}$, the current phasor $\underline{I}$, and the impedance phasor $\underline{Z}$. In figure 5 these phasors are shown.

Fig. 5: phasors of the pure loads

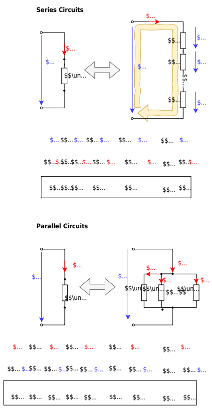


Application on Impedance Networks

Simple Networks

In the chapter Kirchhoff's Circuit Laws we already had a look at simple networks like a series or parallel circuit of resistors.
These formulas not only apply to ohmic resistors but also to impedances:

Fig. 6: Simple Networks

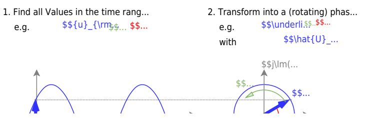


Similarly, the voltage divider, the current divider, the star-delta transformation, and the Thevenin and Northon Theorem can be used, by substituting resistances with impedances. This means for example, every linear source can be represented by an output impedance $\underline{Z}_o$ and an ideal voltage source $\underline{U}$.

More "complex" Networks

For more complex problems having AC values in circuitries, the following approach is beneficial. This concept will be used in the next chapter and in circuit design.

Fig. 7: Approach for AC cicruitries



Notice:

For a complex number are always two values are needed. These are either

1. the real part (e.g. the resistance) and the imaginary part (e.g. the reactance), or
2. the absolute value (e.g. the absolute value of the impedance) and the phase

Therefore, instead of the form $\underline{Z}=Z\cdot {\rm e}^{{\rm j}\varphi}$ for the phasors often the form $Z\angle{\varphi}$ is used.

Common pitfalls

- ...

Exercises

Worked examples

Exercise 6.5.1 Two voltage sources

Two ideal AC voltage sources 1 and 2 shall generate the RMS voltage drops $U_1 = 100~\rm V$ and $U_2 = 120~\rm V$.
The phase shift between the two sources shall be $+60°$. The phase of source 1 shall be $\varphi_1=0°$.
The two sources shall be located in series.

1. Draw the phasor diagram for the two voltage phasors and the resulting phasor.

   Solution 1

   draw.io Start drawing by clicking here

   The phasor diagram looks roughly like this:

2. Calculate the resulting voltage and phase.

   Solution 2

   By the law of cosine, we get: \begin{align*} U&= \sqrt{{{U_1 }^2}+{{U_1 }^2}-2{U_1 } \cdot{U_2 }\cdot \cos(180°- {\varphi_2})} \\ &= \sqrt{{{100~V}^2}+{{120~V}^2}-2\cdot{100~V} \cdot{120~V}\cdot \cos(120°)} \end{align*}

   The angle is by the tangent of the relation of the imaginary part to the real part of the resulting voltage. \begin{align*} \varphi&= \arctan 2 \left(\frac{Im\{\underline{U}\}}{Re\{\underline{U}\}} \right)\\ &= \arctan 2 \left(\frac{Im\{\underline{U}_1\}+Im\{\underline{U}_2\}}{Re\{\underline{U}_1\}+\{\underline{U}_2\}} \right)\\ &= \arctan 2 \left(\frac{{U}_2 \cdot\sin(\varphi_2)}{{U}_1 +{U}_2\cos(\varphi_2)} \right)\\ &= \arctan 2 \left(\frac{120~{\rm V}\cdot\sin(60°)}{100~{\rm V}+120~V\cos(60°)} \right) \end{align*}

   Final value

   \begin{align*} U &=190.79~{\rm V} \\ \varphi&=33° \end{align*}

3. Is the resulting voltage the RMS value or the amplitude?

   Solution 3

   The resulting voltage is the RMS value.

The source 2 shall now be turned around (the previous plus pole is now the minus pole and vice versa).

4. Draw the phasor diagram for the two voltage phasors and the resulting phasor for the new circuit.

   Solution 4

   The phasor diagram looks roughly like this.

Start drawing by clicking here

But have a look at the solution for question 5!

5. Calculate the resulting voltage and phase.

Solution 5

By the law of cosine, we get: \begin{align*} U&= \sqrt{{{U_1 }^2}+{{U_1 }^2}-2{U_1 } \cdot{U_2 }\cdot \cos(180°- {\varphi_1})} &= \sqrt{{{100~V}^2}+{{120~V}^2}-2\cdot{100~V} \cdot{120~V}\cdot \cos(60°)} \end{align*} The angle is by the tangent of the relation of the imaginary part to the real part of the resulting voltage. \begin{align*} \varphi&= \arctan 2 \left(\frac{Im\{\underline{U}\}} {Re\{\underline{U}\}} \right)\\ &= \arctan 2 \left(\frac{Im\{\underline{U}_1\}+Im\{\underline{U}_2\}}{Re\{\underline{U}_1\}+\{\underline{U}_2\}} \right)\\ &= \arctan 2 \left(\frac{-{U}_2 \cdot\sin(\varphi_2)}{{U}_1 -{U}_2\cos(\varphi_2)} \right)\\ &= \arctan 2 \left(\frac{-120~{\rm V}\cdot\sin(60°) }{100~{\rm V}-120~V\cos(60°)} \right) \\ &= \arctan 2 \left(\frac{-103.92...~{\rm V} }{+40~{\rm V}} \right) \end{align*} The calculated (positive) horizontal and (negative) vertical dimension for the voltage indicates a phasor in the fourth quadrant. Does it seem right?
The phasor diagram which was shown in answer 4. cannot be correct.
With the correct lengths and angles, the real phasor diagram looks like this:

Start drawing by clicking here

Here the phasor is in the fourth quadrant with a negative angle.

Final value

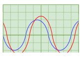
\begin{align*} U &=111.355~{\rm V}\\ \varphi&=-68.948° \end{align*}

Notice:

Be aware that some of the calculators only provide $\tan^{-1}$ or $\arctan$ and not $\arctan2$!
Therefore, you have always to check whether the solution lies in the correct quadrant.

Exercise 6.5.2 oscilloscope plot

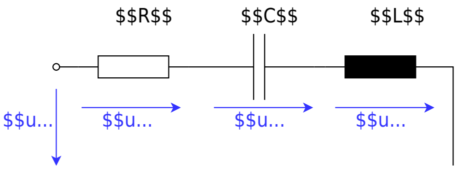
The following plot is visible on an oscilloscope (= plot tool for voltages and current).



1. What is the RMS value of the current and the voltage? What is the frequency f and the phase φ? Does the component under test behave ohmic, capacitive, or inductive?
2. How would the equivalent circuit look like, when it is built by two series components?
3. Calculate the equivalent component values (R, C or L) of the series circuit.
4. How would the equivalent circuit look like, when it is built by two parallel components?
5. Calculate the equivalent component values (R, C or L) of the parallel circuit.

Exercise 6.5.3 Series Circuit

The following circuit shall be given.



This circuit is used with different component values, which are given in the following. Calculate the RMS value of the missing voltage and the phase shift φ between U and I.

1. $U_R = 10~\rm V$, $U_L = 10~\rm V$, $U_C = 20~\rm V$, $U=\rm ?$

   Solution

   The drawing of the voltage pointers is as follows:

The voltage U is determined by the law of Pythagoras \begin{align*} U &= \sqrt{{{U_R } ^2}+{({U_L }-{U_C }})^2} \\ &= \sqrt{(10~{\rm V})^2+ {({10~{\rm V}}-{20~{\rm V}}})^2} \end{align*} The phase shift angle is calculated by simple geometry. \begin{align*} \tan(\varphi)&=\frac{{U_L }-{U_C }}{U_R}\\ &=\frac{{10~{\rm V}}-{20~{\rm V}}}{10~{\rm V}} \end{align*} Considering that the angle is in the fourth quadrant we get:

Final value

\begin{equation*} U=\sqrt{2}\cdot 10~{\rm V} = 14.1~{\rm V} \qquad \varphi=-45° \end{equation*}

2. $U_R = ?$, $U_L = 150~\rm V$, $U_C = 110~\rm V$, $U=50~\rm V$

Solution 2

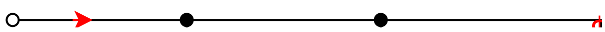
The drawing of the voltage pointers is as follows:

The voltage U_R is determined by the law of Pythagoras \begin{align*} U_R&=\sqrt{{U ^2}+{({U_L} -{U_C}})^2}\\ &=\sqrt{(50~\rm V)^2 +{(150~\rm V -110~\rm V})^2} \end{align*} The phase shift angle is calculated by simple geometry. \begin{align*} \tan(\varphi)&=\frac{{U_L} -{U_C} }{U_R}\\ &=\frac{{150~\rm V}-{110~\rm V}}{30~\rm V} \end{align*} Considering that the angle is in the fourth quadrant we get:

Final value

\begin{equation*} U_R= 30~{\rm V}\qquad \varphi=53.13° \end{equation*}

Exercise 6.5.4 Parallel Circuit

The following circuit shall be given. 

in the following, some of the numbers are given. Calculate the RMS value of the missing currents and the phase shift φ between U and I.

1. $I_R = 3~\rm A$, $I_L = 1 ~\rm A$, $I_C = 5 ~\rm A$, $I=?$
2. $I_R = ?$, $I_L = 1.2~\rm A$, $I_C = 0.4~\rm A$, $I=1~\rm A$

Exercise 6.5.5 Complex Calculation I

The following two currents with similar frequencies, but different phases have to be added. Use complex calculation!

- $i_1(t) = \sqrt{2} \cdot 2 ~A \cdot \cos (\omega t + 20°)$
- $i_2(t) = \sqrt{2} \cdot 5 ~A \cdot \cos (\omega t + 110°)$

Exercise 6.5.6 Complex Calculation II

Two complex impedances $\underline{Z}_1$ and $\underline{Z}_2$ are investigated. The resulting impedance for a series circuit is $60~\Omega + \rm j \cdot 0 ~\Omega $. The resulting impedance for a parallel circuit is $25~\Omega + \rm j \cdot 0 ~\Omega $.

Result

What are the values for $\underline{Z}_1$ and $\underline{Z}_2$?
\begin{align*} R_1 &= 30~\Omega \\ R_2 &= 30~\Omega \\ X_1 &= \mp \sqrt{600}~\Omega \approx \mp 24.5~\Omega \\ X_2 &= \pm \sqrt{600}~\Omega \approx \pm 24.5~\Omega \\ \end{align*}

Solution

It's a good start to write down all definitions of the given values:

- the given values for the series circuit ($\square_\rm s$) and the parallel circuit ($\square_\rm p$) are: \begin{align*} R_\rm s = 60 ~\Omega , \quad X_\rm s = 0 ~\Omega \\ R_\rm p = 25 ~\Omega , \quad X_\rm p = 0 ~\Omega \\ \end{align*}
- the series circuit and the parallel circuit results into: \begin{align*} R_{\rm s} = \underline{Z}_1 + \underline{Z}_2 \tag{1} \\ R_{\rm p} = \underline{Z}_1 || \underline{Z}_2 \tag{2} \\ \end{align*}
- the unknown values of the two impedances are: \begin{align*} \underline{Z}_1 = R_1 + {\rm j}\cdot X_1 \tag{3} \\ \underline{Z}_2 = R_2 + {\rm j}\cdot X_2 \tag{4} \\ \end{align*}

Based on (1),(3) and (4): \begin{align*} R_\rm s &= \underline{Z}_1 &&+ \underline{Z}_2 \\ &= R_1 + {\rm j}\cdot X_1 &&+ R_2 + {\rm j}\cdot X_2 \\ \rightarrow 0 &= R_1 + R_2 - R_\rm s &&+ {\rm j}\cdot (X_1 + X_2) \\ \end{align*}
Real value and imaginary value must be zero: \begin{align*} R_1 &= R_{\rm s} - R_2 \tag{5} \\ X_1 &= - X_2 \tag{6} \end{align*}

Based on (2) with $R_\rm s = \underline{Z}_1 + \underline{Z}_2$ (1): \begin{align*} R_{\rm p} &= {{\underline{Z}_1 \cdot \underline{Z}_2}\over{\underline{Z}_1 + \underline{Z}_2}} \\ &= {{\underline{Z}_1 \cdot \underline{Z}_2}\over{R_\rm s}} \\ \\ R_{\rm p} \cdot R_{\rm s} &= \underline{Z}_1 \cdot \underline{Z}_2 \\ &= (R_1 + {\rm j}\cdot X_1)\cdot (R_2 + {\rm j}\cdot X_2) \\ &= R_1 R_2 + {\rm j}\cdot (R_1 X_2 + R_2 X_1) - X_1 X_2 \\ \end{align*}

Substituting R_1 and X_1 based on (5) and (6): \begin{align*} R_{\rm p} \cdot R_{\rm s} &= (R_{\rm s} - R_2) R_2 + {\rm j}\cdot ((R_{\rm s} - R_2) X_2 - R_2 X_2) + X_2 X_2 \\ \rightarrow 0 &= R_{\rm s} R_2 - R_2^2 + X_2^2 - R_{\rm p} \cdot R_{\rm s} + {\rm j}\cdot ((R_{\rm s} - R_2) X_2 - R_2 X_2) \\ \end{align*}

Again real value and imaginary value must be zero: \begin{align*} 0 &= j\cdot ((R_{\rm s} - R_2) X_2 - R_2 X_2) \\ &= R_{\rm s}X_2 - 2 \cdot R_2 X_2 \\ \rightarrow R_2 = {{1}\over{2}} R_{\rm s} \tag{7} \\ \\ 0 &= R_{\rm s} R_2 - R_2^2 + X_2^2 - R_{\rm p} \cdot R_{\rm s} \\ &= R_{\rm s} ({{1}\over{2}} R_{\rm s}) - ({{1}\over{2}} R_{\rm s})^2 - X_2^2 - R_{\rm p} \cdot R_{\rm s} \\ &= {{1}\over{4}} R_{\rm s}^2 + X_2^2 - R_{\rm p} \cdot R_{\rm s} \\ \rightarrow X_2 = \pm \sqrt{R_{\rm p} \cdot R_{\rm s} - {{1}\over{4}} R_{\rm s}^2 } \tag{8} \\ \\ \end{align*}

The concluding result is: \begin{align*} (5)+(7): \quad R_1 &= {{1}\over{2}} R_{\rm s} \\ (7): \quad R_2 &= {{1}\over{2}} R_{\rm s} \\ (6)+(8) \quad X_1 &= \mp \sqrt{R_{\rm p} \cdot R_{\rm s} - {{1}\over{4}} R_{\rm s}^2 } \\ (8): \quad X_2 &= \pm \sqrt{R_{\rm p} \cdot R_{\rm s} - {{1}\over{4}} R_{\rm s}^2 } \end{align*}

Exercise 6.5.7 real Coils I

A real coil has both ohmic and inductance behavior. At DC voltage the resistance is measured as $9 ~\Omega$. With an AC voltage of $5~\rm V$ at $50~\rm Hz$ a current of $0.5~\rm A$ is measured.

What is the value of the inductance L?

Exercise 6.5.8 real Coils II

A real coil has both ohmic and inductance behavior. This coil has at $100~\rm Hz$ an impedance of $1.5~\rm k\Omega$ and a resistance $1~\rm k\Omega$.

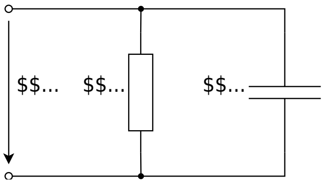
What is the value of the reactance and inductance?

Exercise 6.5.9 Capacitors and Resistance I

An ideal capacitor is in series with a resistor $R=1~\rm k\Omega$. The capacitor shows a similar voltage drop to the resistor for $100~\rm Hz$.

What is the value of the capacitance?

Exercise E1 EMI Input Check of an AC Power Module: Parallel Resistor-Capacitor Branch



An AC power module contains a damping resistor and an EMI capacitor connected in parallel at its input. For commissioning, the complex input behavior at mains frequency shall be evaluated.

Data: \begin{align*} u_1 &= \hat U_1 \cos(\omega t) \\ \hat U_1 &= 325~{\rm V} \\ f &= 50~{\rm Hz} \\ R &= 220~{\rm \Omega} \\ C &= 4.7~{\rm \mu F} \end{align*}

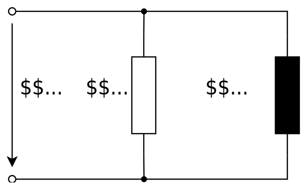
1. How large is ωC?

2. Calculate the complex input impedance.

3. Calculate the input current by magnitude and phase.

4. Draw the phasor diagram for all currents and voltages.

Exercise E2 AC Magnet Valve Branch: Parallel Resistor-Inductor Circuit



An AC control unit contains a magnetizing branch modeled as a resistor in parallel with an inductor. The complex input behavior at operating frequency shall be determined.

Data: \begin{align*} u_1 &= \hat U_1 \cos(\omega t) \\ \hat U_1 &= 170~{\rm V} \\ f &= 60~{\rm Hz} \\ R &= 220~{\rm \Omega} \\ L &= 325~{\rm mH} \end{align*}

1. How large is ωL?

2. Calculate the complex input impedance.

3. Calculate the input current by magnitude and phase.

4. Draw the phasor diagram for all currents and voltages.

Exercise E3 AC Sensor Front-End: Composite Reactive Network

A sensor front-end in an industrial AC cabinet is modeled by a series capacitor, a series inductance, and a parallel output branch consisting of a resistor and another inductance. The complex input behavior shall be analyzed.

Data:

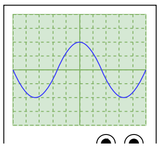
$$\begin{align*} \omega C &= 0.01~{\rm S} \\ \omega L_1 &= 50~{\rm \Omega} \\ \omega L_2 &= 200~{\rm \Omega} \\ R &= 100~{\rm \Omega} \\ \hat U_1 &= 325~{\rm V} \end{align*}$$

1. Determine the complex input impedance of the circuit.

2. Calculate the input current by magnitude and phase.

3. Draw the phasor diagram for all currents and voltages.

Exercise E4 Compensated Oscilloscope Probe on a Long Cable



A sinusoidal measurement signal shall be observed through a long cable and an oscilloscope probe. The oscilloscope input is characterized by the input resistance R_E and input capacitance C_E. The cable is represented by its capacitance C_K. The probe itself consists of a resistor R_T in parallel with an adjustable capacitor C_T.

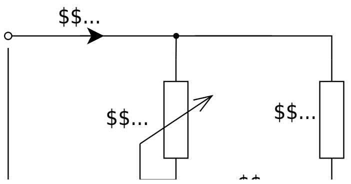
1. Give the complex impedance of the probe.

2. Draw the circuit diagram for the given arrangement.

3. What is the ratio between measured voltage and oscilloscope input voltage, $v_U=\hat U_M/\hat U_E$, as a function of the given resistances and capacitances?

4. How must the probe capacitor C_T be adjusted so that v_U becomes frequency-independent? What is v_U under this condition?

Exercise E5 Phase-Shift Bridge for Trigger and Synchronization Signals



A phase-shift bridge is used in a control cabinet to generate a shifted AC reference signal for triggering and synchronization. One branch contains a potentiometer R_P and a capacitor, while the other branch is a symmetric resistor divider. The output voltage u_2 is taken between the two center nodes.

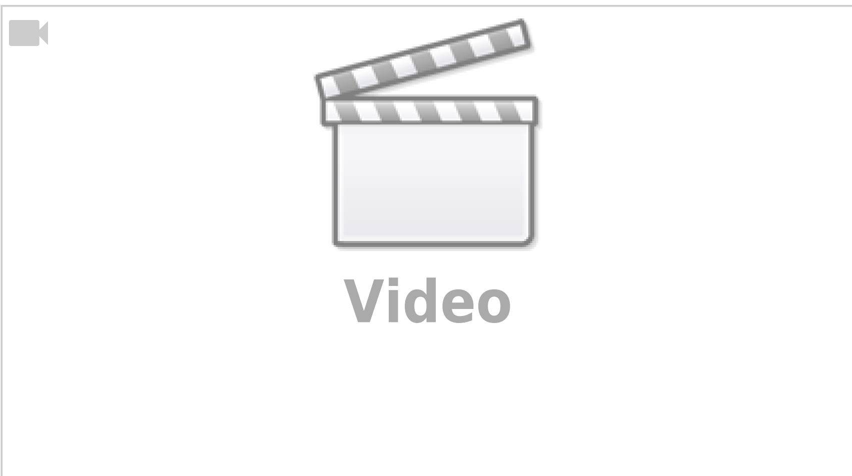
Data: \begin{align*} R &= 1~{\rm k\Omega} \\ \omega C &= 1~{\rm mS} \\ \hat U_1 &= 325~{\rm V} \\ f &= 50~{\rm Hz} \end{align*}

1. The potentiometer is set to $R_P=R$. Draw the phasor diagram for all currents and voltages. Give amplitude and phase of u_2.

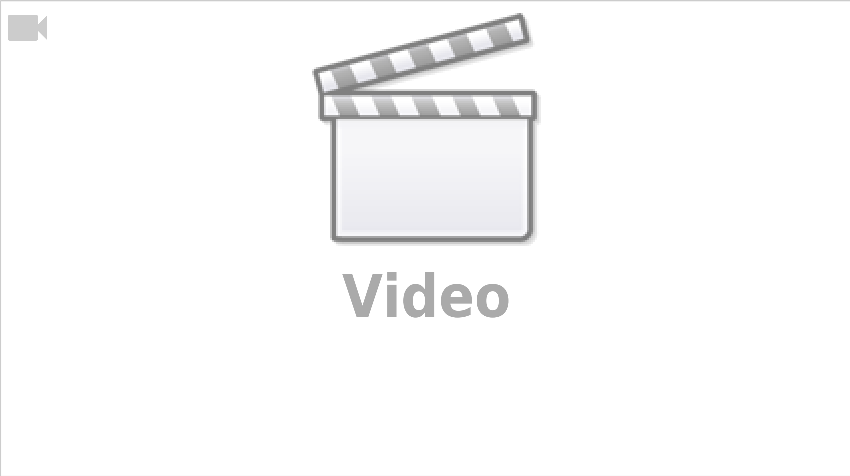
2. How must R_P be adjusted so that u_2 lags by 45°?

3. What is the complex phasor $\underline{\hat U}_2$ as a function of the resistance R_P?

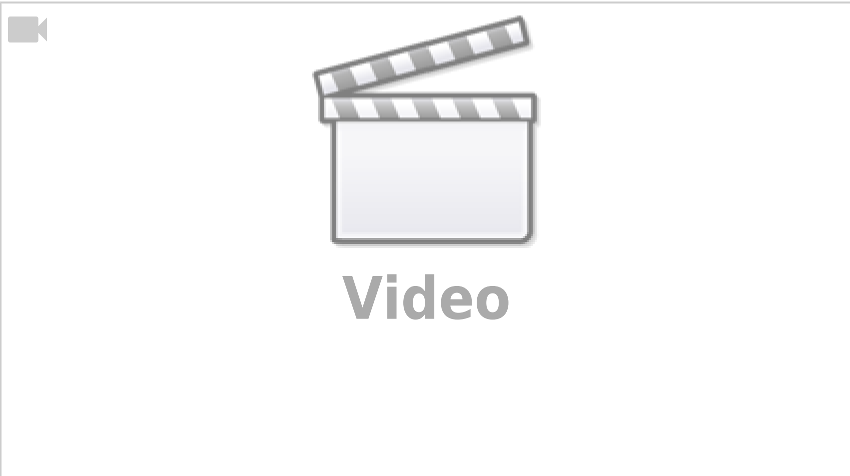
Exercise 6.6.1 Impedance in Series Circuit of multiple Components I



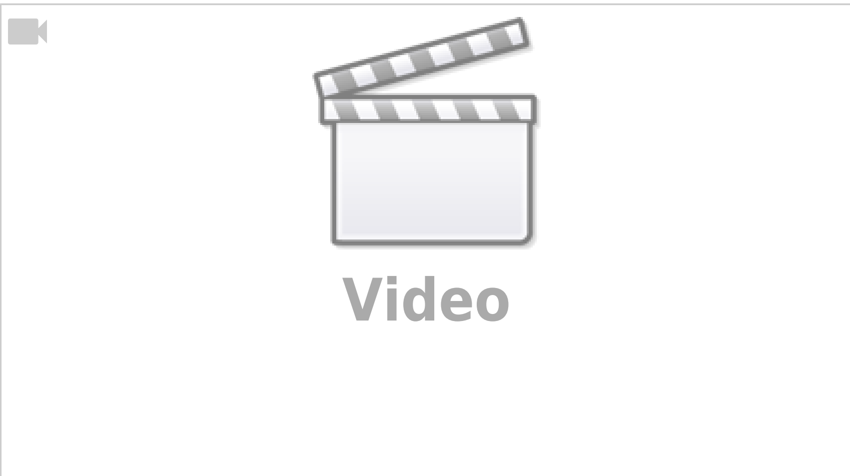
Exercise 6.6.2 Impedance in Series Circuit of multiple Components II



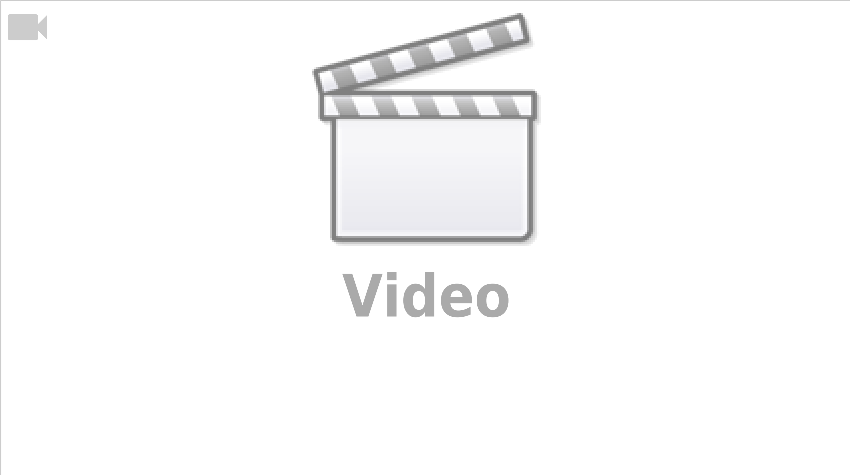
Exercise 6.6.3 Impedance in Parallel Circuit of multiple Components I



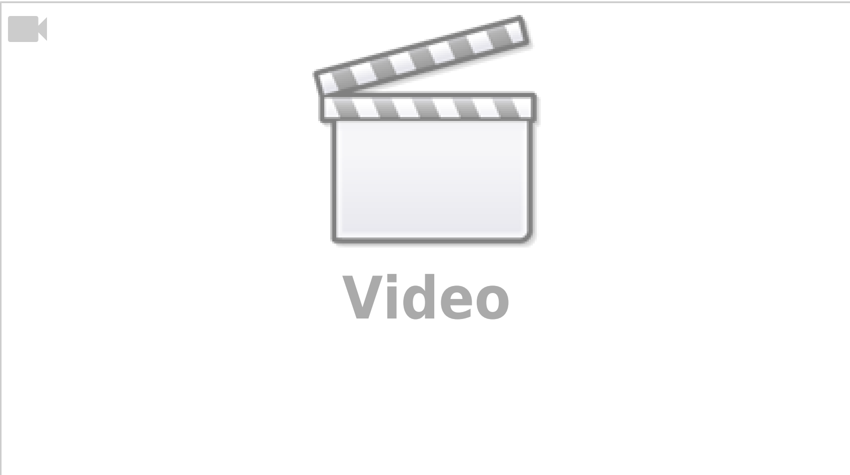
Exercise 6.6.4 Impedance in Mixed Parallel and Series Circuit of multiple Components I



Exercise 6.6.5 Impedance in Mixed Parallel and Series Circuit of multiple Components II

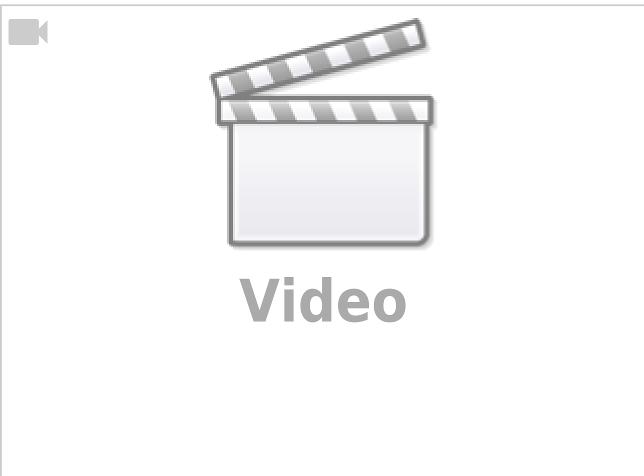


Exercise 6.6.6 Impedance in Mixed Parallel and Series Circuit of multiple Components III

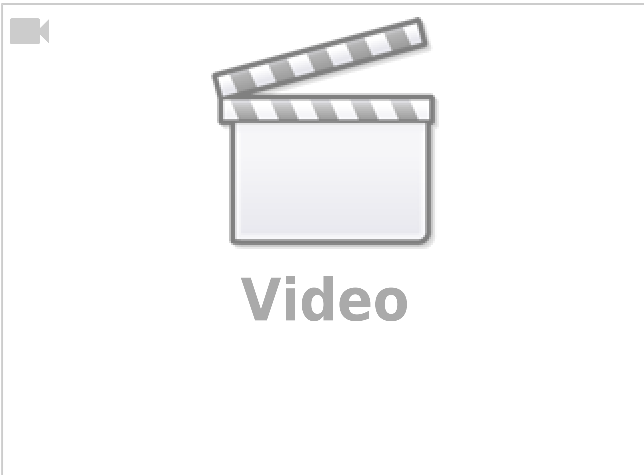


Embedded resources

The following two videos explain the basic terms of the complex AC calculus: Impedance, Reactance, Resistance



This does the same



[1] in general, here the integration constant must be conside

red. This is however often neglectable since only AC values (without a DC value) are considered.