Block 09/10 — Transformers and Magnetic Coupling

Student Group

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Learning objectives

> After this 90-minute block, you can
>
> - explain how two coils can exchange energy by a common magnetic flux \(\Phi\).
> - use the ideal transformer equations
>
> \[\begin{align*} \frac{\underline{U}_1}{\underline{U}_2}=\frac{N_1}{N_2}=n, \qquad \frac{\underline{I}_1}{\underline{I}_2}=-\frac{1}{n} \end{align*} \] with a clear sign convention.
>
> - explain mutual inductance \(M\) using flux linkage and magnetic reluctance \(R_{\rm m}\).
> - distinguish **main flux**, **leakage flux**, **copper losses**, and **iron losses** in a real transformer.
> - refer secondary-side quantities to the primary side using \(\underline{U}'_2=n\underline{U}_2\), \(\underline{I}'_2=\frac{1}{n}\underline{I}_2\), \(R'_2=n^2R_2\), and \(X'_{2\sigma}=n^2X_{2\sigma}\).
> - interpret the no-load test and short-circuit test using the reduced equivalent circuit.
> - calculate short-circuit voltage \(u_{\rm k}\), continuous short-circuit current \(I_{\rm 1k}\), and an estimated initial peak short-circuit current.
> - connect transformer parameters to engineering applications in mechatronics and robotics, such as isolated power supplies, motor current measurement, welding transformers, and safety transformers.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Repeat the EEE1 ideas of magnetic flux and induction, magnetic circuits, and inductance and magnetic energy.
- Repeat from EEE2 the use of sinusoidal quantities, complex calculation, and complex power.

For checking your understanding please do the quick checks in the exercise section.

90-minute plan

- **Warm-up (10 min):**
    - Where do transformers occur in robots and automation systems?
    - Recall: Faraday induction from EEE1 — a changing magnetic flux induces a voltage.
    - Recall: in AC analysis we use RMS phasors \(\underline{U}\), \(\underline{I}\), and impedances \(j\omega L\).

- **Core concepts and derivations (55 min):**
    - Ideal transformer: common flux, voltage ratio, current ratio, power balance.
    - Mutual inductance: how flux from one coil links another coil.
    - Magnetic coupling with reluctance $R_{\rm m}$.
    - Real transformer: winding resistances, leakage inductances, iron-loss resistance.
    - Reduced equivalent circuit: refer secondary quantities to the primary side.
    - No-load and short-circuit operation: what can be measured, what can be neglected.
- **Practice (20 min):**
    - Quick ratio calculations for step-up and step-down transformers.
    - Unit checks for $j\omega L$, $j\omega N\Phi$, and $u_{\rm k}$.
    - Short-circuit current calculation for a transformer used in an actuator supply.
- **Wrap-up (5 min):**
    - Summary box: ideal transformer, mutual inductance, real transformer, reduced circuit, short-circuit parameters.
    - Common pitfalls checklist.

Conceptual overview

- A transformer is **not** a DC component. It needs a changing magnetic flux. In normal operation this is usually a sinusoidal flux created by AC voltage.
- The transformer does not “create power”. Ideally, it trades voltage for current:

$$\begin{align*} \text{higher voltage} \quad \Longleftrightarrow \quad \text{lower current} \end{align*}$$

- The link between the two windings is the magnetic field in the iron core. This continues directly from EEE1:
    - induction explains why a changing flux induces voltage.
    - magnetic circuits explains why the iron core guides the flux.
    - inductance explains how flux linkage and current are connected.
- Mutual inductance M measures how strongly one coil “notices” the changing current in another coil.
- A real transformer is almost ideal, but not quite:
    - R_1, R_2: copper losses in the windings.
    - $L_{1\sigma}, L_{2\sigma}$: leakage flux that does not couple both windings.
    - $R_{\rm Fe}$: iron losses in the core.
    - $L_{\rm H}$: main magnetizing inductance needed to create the main flux.
- In engineering, transformer data such as $u_{\rm k}$ are not abstract: they determine voltage drop, fault current, thermal stress, and protection design.

Core content

Mutual induction: the key idea before the transformer

Short Review of the Flux

In EEE1 we considered magnetic flux Φ, flux linkage / linked flux Ψ, and induction.
For one coil with N turns the flux linkage is

$$\begin{align*} \Psi = N\Phi . \end{align*}$$

Faraday's law gives

$$\begin{align*} u(t)=\frac{{\rm d}\Psi}{{\rm d}t}=N\frac{{\rm d}\Phi}{{\rm d}t}. \end{align*}$$

(Be aware of Lenz law, whenever you want to draw the votage arrows)
In sinusoidal steady state this becomes the phasor equation

$$\begin{align*} \underline{U}=j\omega\underline{\Psi}=j\omega N\underline{\Phi}. \end{align*}$$

This is the starting point for the transformer.

Now, we look onto the situation of two coils nearby each other and expand this formula for the induced voltage.
For this, we see:

> A changing current in coil 1 creates a changing magnetic flux.
> If part of this flux passes through coil 2, a voltage is induced in coil 2.
> This is called **mutual induction**.

Fig. 4: Mutual induction of two coils: only part of the flux created by coil 1 links coil 2.

The flux created by coil 1 can be split into

$$\begin{align*} \Phi_{11} = {\color{blue}{\Phi_{21}}} + {\color{orange}{\Phi_{\rm S1}}}. \end{align*}$$

- Φ_{11}: total flux created by coil 1.
- ${\color{blue}{\Phi_{21}}}$: part of this flux that also links coil 2.
- ${\color{orange}{\Phi_{\rm S1}}}$: stray or leakage flux that does **not** link coil 2.

The voltage induced in coil 2 is

$$\begin{align*} u_{{\rm ind},2}(t) = N_2\frac{{\rm d}{\color{blue}{\Phi_{21}}}}{{\rm d}t}. \end{align*}$$

Analogies

Analogy 1: two pendulums connected by a spring

Imagine two pendulums connected by a weak spring.

- If pendulum \(1\) moves, the spring can make pendulum \(2\) move as well.
- A strong spring transfers the motion strongly.
- A weak spring transfers the motion only weakly.
- If the spring is missing, pendulum \(2\) does not react.

For coupled coils:

- the changing motion corresponds to changing current,
- the spring corresponds to the magnetic coupling,
- the motion transferred to the second pendulum corresponds to the induced voltage,
- weak coupling means that only a small part of the magnetic flux links both coils.

Analogy 2: a leaky magnetic pipe

The magnetic core can be imagined as a pipe guiding magnetic flux.

- A good iron core is like a wide, low-resistance pipe: most flux reaches the second coil.
- A large air gap is like a narrow, difficult path: less flux reaches the second coil.
- Leakage flux is like flow escaping through side paths: it belongs to the first coil but does not help the second coil.

This image is helpful for transformers, wireless charging coils, and current sensors.

Engineering examples

- **Transformer:** very strong coupling because the iron core guides most of the flux through both windings.
- **Wireless charger:** weaker coupling because the flux must cross an air gap and the coils may be misaligned.
- **Current transformer:** the measured conductor acts like a one-turn primary winding; the secondary winding detects the changing magnetic field.
- **Relay coil near signal wiring:** unwanted coupling can induce noise voltages in nearby loops.

Linked fluxes and mutual inductance

For a single coil we already know that its flux linkage $\Psi = N\Phi$ is proportional to the current i through the coil

\[\begin{align*} \Psi=L i . \end{align*} \]

For two coupled coils 1 and 2, each flux linkage Ψ_1 and Ψ_2 can depend on both currents i_1 and i_2.
Not only the current through the coil generates a part of the flux linkage, but also the other coil provides a part for the flux linkage.

\[\begin{align*} \Psi_1 &= \underbrace{{\color{green}{L_{11}i_1}}}_{\text{self-linkage of coil 1}}

+ \underbrace{{\color{blue}{M_{12}i_2}}}_{\text{mutual linkage from coil 2}}, \\[4pt] \Psi_2 &= \underbrace{{\color{blue}{M_{21}i_1}}}_{\text{mutual linkage from coil 1}} + \underbrace{{\color{green}{L_{22}i_2}}}_{\text{self-linkage of coil 2}}. \end{align*} \]

For most transformer calculations we use the symmetric case of the mutual inductances.
(this is true for passive, stationary, and reciprocal situations, like transformers, but not necessarily for motors or complex setups)

\[\begin{align*} {\color{blue}{M_{12}}}={\color{blue}{M_{21}}}=M. \end{align*} \]

Then

\[\begin{align*} \boxed{ \begin{pmatrix} \Psi_1\\ \Psi_2 \end{pmatrix} = \begin{pmatrix} {\color{green}{L_{11}}} & {\color{blue}{M}}\\ {\color{blue}{M}} & {\color{green}{L_{22}}} \end{pmatrix} \begin{pmatrix} i_1\\ i_2 \end{pmatrix} } \end{align*} \]

and

\[\begin{align*} M=k\sqrt{L_{11}L_{22}}. \end{align*} \]

Here \(k\) is the coupling coefficient.

Coupling coefficient	Interpretation	Typical example
\(k=0\)	no useful flux from one coil links the other coil	coils far apart
\(0<k<1\)	partial coupling	wireless charger with air gap or misalignment
\(k\approx 1\)	almost all useful flux links both coils	transformer with iron core
sign of \(k\)	depends on winding direction and reference arrows	dot convention (see below)

Tab. 1: Meaning of the coupling coefficient \(k\)

The mutual inductance \(M\) answers the question:

> How much flux linkage appears in coil \(2\) when the current in coil \(1\) changes?

- A large \(M\) means strong interaction.
- A small \(M\) means weak interaction.

Engineering example: wireless charging

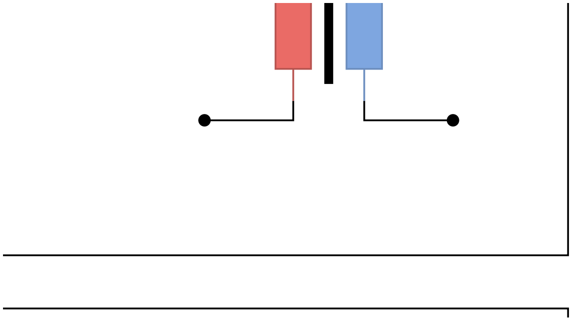
In wireless charging, the transmitter coil and receiver coil are separated by an air gap. The coupling coefficient \(k\) is much smaller than in a transformer with an iron core.

If the receiver is misaligned, less flux from the transmitter passes through it. Then \(M\) decreases, the induced voltage decreases, and the transmitted power decreases.

Polarity and the dot convention

The sign of the mutual term depends on the winding direction and on the chosen current reference arrows.

Fig. 1: Dot convention: the dots indicate corresponding winding ends.



> **Rule of thumb**
>
> - If both currents enter dotted terminals, the mutual fluxes support each other.
> - If one current enters a dotted terminal and the other current leaves a dotted terminal, the mutual fluxes oppose each other.

Fig. 2: Positive coupling: currents enter corresponding dotted terminals.

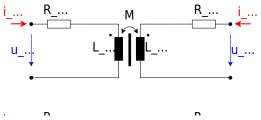
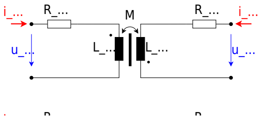


Fig. 3: Negative coupling: only one current enters a dotted terminal.



For positive coupling:

\[\begin{align*} u_1 &= {\color{green}{L_{11}\frac{{\rm d}i_1}{{\rm d}t}}} + {\color{blue}{M\frac{{\rm d}i_2}{{\rm d}t}}}, \\[4pt] u_2 &= {\color{blue}{M\frac{{\rm d}i_1}{{\rm d}t}}} + {\color{green}{L_{22}\frac{{\rm d}i_2}{{\rm d}t}}}. \end{align*} \]

For negative coupling, the sign of the \(M\)-term changes in the chosen equation system.

Tunnel Analogy

The dots are like matching openings for magnetic action.
A positive current (e.g. i_1) entering the dotted terminal of one winding produces a positive induced voltage (e.g. aligned with u_2) at the dotted terminal of the other winding.
With only a load R_2 connected to the secondary side, this voltage tends to drive current out of the dotted terminal into the load (i_2 has to be inverted, since the transformer is a source then).

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Common pitfalls

- …

Exercises

Worked examples

…

Embedded resources

Explanation (video): …