Block 09/10 — Transformers and Magnetic Coupling

Student Group

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Learning objectives

After this 90-minute block, you can

- explain how two coils can exchange energy by a common magnetic flux Φ.
- use the ideal transformer equations

$$\begin{align*} \frac{\underline{U}_1}{\underline{U}_2}=\frac{N_1}{N_2}=n, \qquad \frac{\underline{I}_1}{\underline{I}_2}=-\frac{1}{n} \end{align*}$$

with a clear sign convention.

- explain mutual inductance M using flux linkage and magnetic reluctance $R_{\rm m}$.
- distinguish **main flux**, **leakage flux**, **copper losses**, and **iron losses** in a real transformer.
- refer secondary-side quantities to the primary side using $\underline{U}'_2=n\underline{U}_2$, $\underline{I}'_2=\frac{1}{n}\underline{I}_2$, $R'_2=n^2R_2$, and $X'_{2\sigma}=n^2X_{2\sigma}$.
- interpret the no-load test and short-circuit test using the reduced equivalent circuit.
- calculate short-circuit voltage $u_{\rm k}$, continuous short-circuit current $I_{\rm 1k}$, and an estimated initial peak short-circuit current.
- connect transformer parameters to engineering applications in mechatronics and robotics, such as isolated power supplies, motor current measurement, welding transformers, and safety transformers.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Repeat the EEE1 ideas of magnetic flux and induction, magnetic circuits, and inductance and magnetic energy.
- Repeat from EEE2 the use of sinusoidal quantities, complex calculation, and complex power.

For checking your understanding please do the quick checks in the exercise section.

90-minute plan

- **Warm-up (10 min):**
  - Where do transformers occur in robots and automation systems?
  - Recall: Faraday induction from EEE1 — a changing magnetic flux induces a voltage.
  - Recall: in AC analysis we use RMS phasors $\underline{U}$, $\underline{I}$, and impedances $j\omega L$.

- **Core concepts and derivations (55 min):**
  - Ideal transformer: common flux, voltage ratio, current ratio, power balance.
  - Mutual inductance: how flux from one coil links another coil.
  - Magnetic coupling with reluctance $R_{\rm m}$.
  - Real transformer: winding resistances, leakage inductances, iron-loss resistance.
  - Reduced equivalent circuit: refer secondary quantities to the primary side.
  - No-load and short-circuit operation: what can be measured, what can be neglected.
- **Practice (20 min):**
  - Quick ratio calculations for step-up and step-down transformers.
  - Unit checks for $j\omega L$, $j\omega N\Phi$, and $u_{\rm k}$.
  - Short-circuit current calculation for a transformer used in an actuator supply.
- **Wrap-up (5 min):**
  - Summary box: ideal transformer, mutual inductance, real transformer, reduced circuit, short-circuit parameters.
  - Common pitfalls checklist.

Conceptual overview

- A transformer is **not** a DC component. It needs a changing magnetic flux. In normal operation this is usually a sinusoidal flux created by AC voltage.
- The transformer does not "create power". Ideally, it trades voltage for current:

$$\begin{align*} \text{higher voltage} \quad \Longleftrightarrow \quad \text{lower current} \end{align*}$$

- The link between the two windings is the magnetic field in the iron core. This continues directly from EEE1:
  - induction explains why a changing flux induces voltage.
  - magnetic circuits explains why the iron core guides the flux.
  - inductance explains how flux linkage and current are connected.
- Mutual inductance M measures how strongly one coil "notices" the changing current in another coil.
- A real transformer is almost ideal, but not quite:
  - R_1, R_2: copper losses in the windings.
  - $L_{1\sigma}, L_{2\sigma}$: leakage flux that does not couple both windings.
  - $R_{\rm Fe}$: iron losses in the core.
  - $L_{\rm H}$: main magnetizing inductance needed to create the main flux.
- In engineering, transformer data such as $u_{\rm k}$ are not abstract: they determine voltage drop, fault current, thermal stress, and protection design.

Core content

Short Review of the Flux

In EEE1 we considered magnetic flux Φ, flux linkage / linked flux Ψ, and induction.
For one coil with N turns the flux linkage is

$$\begin{align*} \Psi = N\Phi . \end{align*}$$

Faraday's law gives

$$\begin{align*} u(t)=\frac{{\rm d}\Psi}{{\rm d}t}=N\frac{{\rm d}\Phi}{{\rm d}t}. \end{align*}$$

(Be aware of Lenz law, whenever you want to draw the votage arrows)
In sinusoidal steady state this becomes the phasor equation

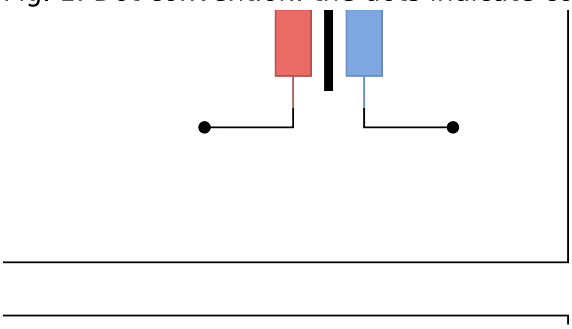
$$\begin{align*} \underline{U}=j\omega\underline{\Psi}=j\omega N\underline{\Phi}. \end{align*}$$

This is the starting point for the transformer.

Polarity and the dot convention

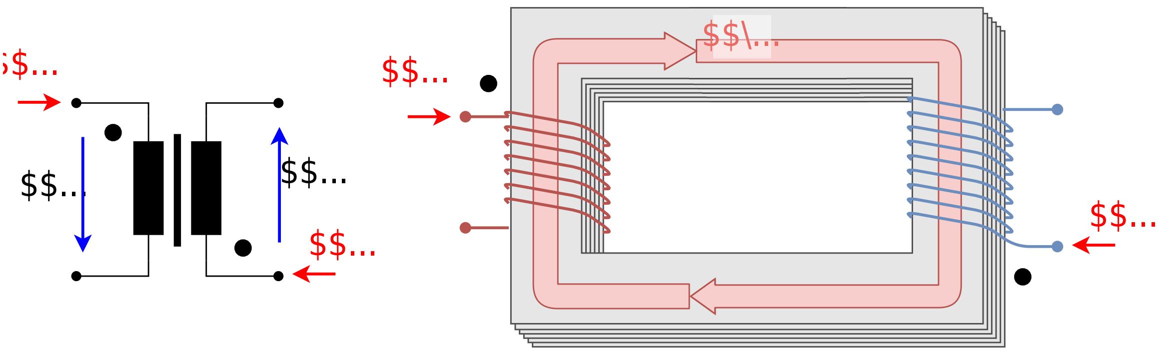
Before we start with the transformer, we have to look on a common convention for the orientation of two coils to each other.

Fig. 1: Dot convention: the dots indicate corresponding winding ends.



> **Rule of thumb**
>
> - If both currents enter dotted terminals, the fluxes support each other.
> - If one current enters a dotted terminal and the other current leaves a dotted terminal, the fluxes oppose each other.

Ideal single-phase transformer



(1) electrically (2) mechanically

For an ideal transformer we assume:

- both windings are linked by the same magnetic flux \(\underline{\Phi} \),
- there is no leakage flux,
- there are no winding resistances,
- there are no iron losses,
- the transformer stores no net energy over one period.

Let \(N_1\) be the number of turns of the primary winding and \(N_2\) the number of turns of the secondary winding.

\[\begin{align*} \underline{\Psi}_1 &= N_1\underline{\Phi}, & \underline{U}_1 &=

j\omega\underline{\Psi}_1 = j\omega N_1\underline{\Phi}, \\ \underline{\Psi}_2 &= N_2\underline{\Phi}, & \underline{U}_2 &= j\omega\underline{\Psi}_2 = j\omega N_2\underline{\Phi}. \end{align*} \]

Dividing the two voltage equations gives the **turns ratio**

$$\begin{align*} \boxed{ \frac{\underline{U}_1^}{\underline{U}_2^} = \frac{N_1}{N_2} = n } \end{align*}$$

with

$$\begin{align*} n=\frac{N_1}{N_2}. \end{align*}$$

Remember: ideal transformer ratios

With the indicated reference arrows and a lossless transformer:

$$\begin{align*} \underline{U}_1\underline{I}_1+\underline{U}_2\underline{I}_2=0 \end{align*}$$

and therefore

$$\begin{align*} \boxed{ \frac{\underline{I}_1}{\underline{I}_2} = -\frac{\underline{U}_2^}{\underline{U}_1^} = -\frac{N_2}{N_1} = -\frac{1}{n} } \end{align*}$$

The minus sign is not a “loss”. It is caused by the chosen current arrows.
The primary side **absorbs** power while the secondary side **delivers** power to the load.

Analogy: gearbox for voltage and current

An ideal transformer behaves like a lossless gearbox:

- a gearbox can trade speed for torque,
- a transformer can trade voltage for current.

For a step-down transformer:

$$\begin{align*} \text{lower voltage} \quad \Longleftrightarrow \quad \text{higher current}. \end{align*}$$

The power is ideally conserved, just as mechanical power is ideally conserved in a lossless gearbox.

Physical interpretation

- If $N_2<N_1$, the transformer steps the voltage down: $U_2<U_1$.
- At the same time, the secondary current can be higher: $I_2>I_1$.
- This is useful in robotics power supplies: a mains-side transformer or isolated converter stage may reduce voltage while increasing available current for actuators.

Exercise E1 step-down transformer for a robot controller

A transformer has $N_1=800$ turns and $N_2=80$ turns. The primary RMS voltage is $U_1=230~{\rm V}$.

$$\begin{align*} n &= \frac{N_1}{N_2} = \frac{800}{80} =10, \\ U_2 &= \frac{U_1}{n} = \frac{230~{\rm V}}{10} =23~{\rm V}. \end{align*}$$

If the secondary side supplies $I_2=4~{\rm A}$, the ideal primary current magnitude is

$$\begin{align*} I_1 &= \frac{I_2}{n} = \frac{4~{\rm A}}{10} =0.4~{\rm A}. \end{align*}$$

The apparent power is equal on both sides:

$$\begin{align*} S_1 &= U_1I_1=230~{\rm V}\cdot 0.4~{\rm A}=92~{\rm VA}, \\ S_2 &= U_2I_2=23~{\rm V}\cdot 4~{\rm A}=92~{\rm VA}. \end{align*}$$

Mutual induction: the key idea for the real transformer

The image of the ideal transformer shall be consecutively developed to a more realistic transformer model.
To do so, we look onto the situation of two coils nearby each other and expand this formula for the induced voltage.
For this, we see:

> A changing current in coil 1 creates a changing magnetic flux.
> (Only) A part of this flux passes through coil 2, a voltage is induced in coil 2.
> This is called **mutual induction**.

Fig. 4: Mutual induction of two coils: only part of the flux created by coil 1 links coil 2.

The flux created by coil 1 can be split into

$$\begin{align*} \Phi_{1\rm H} = {\color{blue}{\Phi_{21}}} + {\color{orange}{\Phi_{\rm \sigma 1}}}. \end{align*}$$

- ${\Phi_{1\rm H}}$: total flux created by coil 1. The 'H' denotes the German word “Haupt” (sometines also given as 'm' for “main”).
- ${\color{blue}{\Phi_{21}}}$: part of this flux that also links coil 2.
- ${\color{orange}{\Phi_{\rm \sigma 1}}}$: stray or leakage flux that does **not** link coil 2. The greek sigma σ is used to depict the term “stray”

For an example, we will have a look onto the instantaneous voltage induced in coil 2:

$$\begin{align*} u_{{\rm ind},2}(t) = \frac{{\rm d}{\color{blue}{\Psi_{21}}}}{{\rm d}t} = N_2\frac{{\rm d}{\color{blue}{\Phi_{21}}}}{{\rm d}t}. \end{align*}$$

The complex voltage induced in coil 2 is

$$\begin{align*} \boxed{ \underline{U}_{{\rm ind},2}(t) = j\omega {\color{blue}{ \underline{\Psi}_{21} }} = j\omega N_2 {\color{blue}{ \underline{\Phi}_{21} }} }. \end{align*}$$

We need these complex represenations for the next steps into the transformer.

1. Mutual induction:
   First, we will investigate combining both fluxes - the flux in a coil generated by itself plus the flux of the other coil
2. Stray flux:
   Then, we add the stray flux to our model
3. Losses:
   Finally, we build a full model including power losses

Analogies

Analogy 1: two pendulums connected by a spring

Imagine two pendulums connected by a weak spring.

- If pendulum 1 moves, the spring can make pendulum 2 move as well.
- A strong spring transfers the motion strongly.
- A weak spring transfers the motion only weakly.
- If the spring is missing, pendulum 2 does not react.

For coupled coils:

- the changing motion corresponds to changing current,
- the spring corresponds to the magnetic coupling,
- the motion transferred to the second pendulum corresponds to the induced voltage,
- weak coupling means that only a small part of the magnetic flux links both coils.

Analogy 2: a leaky magnetic pipe

The magnetic core can be imagined as a pipe guiding magnetic flux.

- A good iron core is like a wide, low-resistance pipe: most flux reaches the second coil.
- A large air gap is like a narrow, difficult path: less flux reaches the second coil.
- Leakage flux is like flow escaping through side paths: it belongs to the first coil but does not help the second coil.

This image is helpful for transformers, wireless charging coils, and current sensors.

Engineering examples

- **Transformer:** very strong coupling because the iron core guides most of the flux through both windings.
- **Wireless charger:** weaker coupling because the flux must cross an air gap and the coils may

be misaligned.
- **Current transformer:** the measured conductor acts like a one-turn primary winding; the secondary winding detects the changing magnetic field.
- **Relay coil near signal wiring:** unwanted coupling can induce noise voltages in nearby loops.

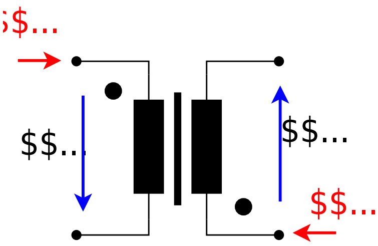
Linked fluxes and mutual inductance

For a single coil we already know that its flux linkage $\Psi = N\Phi$ is proportional to the current i through the coil

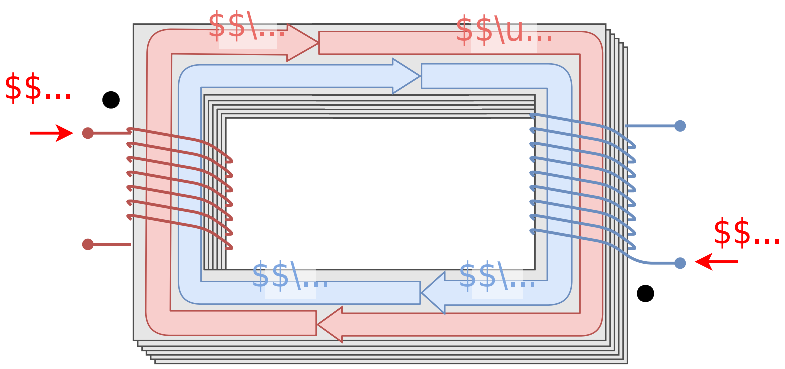
$$\begin{align*} \Psi=L i \end{align*}$$

$$\begin{align*} \underline{\Psi}=L \underline{I} . \end{align*}$$

For two coupled coils 1 and 2, each flux linkage $\underline{\Psi}_1 = N\underline{\Phi}_1$ and $\underline{\Psi}_2 = N\underline{\Phi}_2$ depend on both currents $\underline{I}_1$ and $\underline{I}_2$.



(1) electrically



(2) mechanically

Not only the current through the coil generates a part of the flux linkage, but also the other coil provides a part for the flux linkage.

\[\begin{align*} \underline{\Psi}_1 &= \underbrace{{\color{green}{L_{1\rm H}\underline{I}_1}}}_{\text{self-linkage of coil 1}} + \underbrace{{\color{blue}{M_{12}\underline{I}_2}}}_{\text{mutual linkage from coil 2}}, \\[4pt] \underline{\Psi}_2 &= \underbrace{{\color{blue}{M_{21}\underline{I}_1}}}_{\text{mutual linkage from coil 1}} + \underbrace{{\color{green}{L_{2\rm H}\underline{I}_2}}}_{\text{self-linkage of coil 2}}. \end{align*} \]

For most transformer calculations we use the symmetric case of the mutual inductances.
(this is true for passive, stationary, and reciprocal situations, like transformers, but not necessarily for

motors or complex setups)

$$\begin{align*} {\color{blue}{M_{12}}}={\color{blue}{M_{21}}}={\color{blue}{M}}. \end{align*}$$

Then

$$\begin{align*} \boxed{ \begin{pmatrix} \underline{\Psi}_1\\ \underline{\Psi}_2 \end{pmatrix} = \begin{pmatrix} {\color{green}{L_{1\rm H}}} & {\color{blue}{M}}\\ {\color{blue}{M}} & {\color{green}{L_{2\rm H}}} \end{pmatrix} \begin{pmatrix} \underline{I}_1\\ \underline{I}_2 \end{pmatrix} } \end{align*}$$

and

$$\begin{align*} M=k\sqrt{L_{1\rm H}L_{2\rm H}}. \end{align*}$$

Here k is the coupling coefficient. In the shown transformer k is 1 since all flux from 1 flows through 2 and vice versa.
In reality that is not the case as explained in the next chapters.

Coupling coefficient	**Interpretation**	**Typical example**
$k=0$	no useful flux from one coil links the other coil	coils far apart
$0<k<1$	partial coupling	wireless charger with air gap or misalignment
$k\approx 1$	almost all useful flux links both coils	transformer with iron core
sign of k	depends on winding direction and reference arrows	dot convention (see below)

Tab. 1: Meaning of the coupling coefficient k

The mutual inductance M answers the question:

> How much flux linkage appears in coil 2 when the current in coil 1 changes?

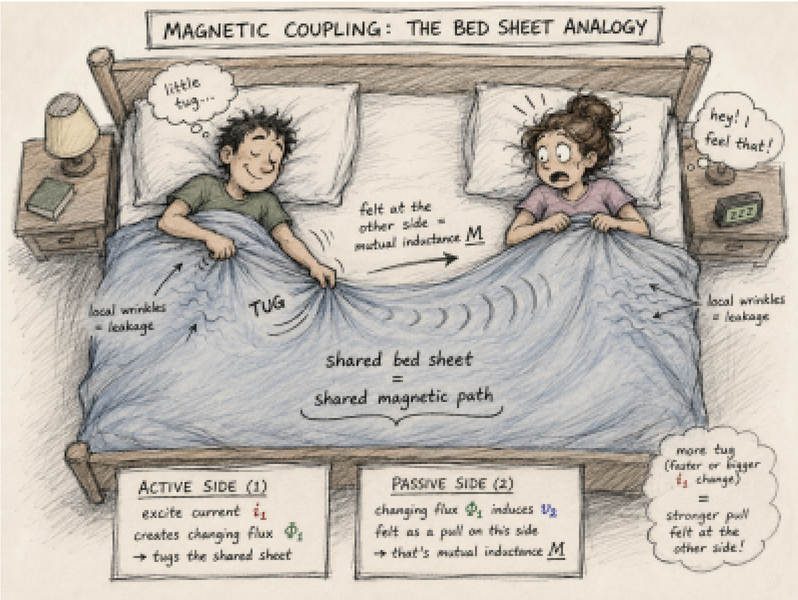
- A large M means strong interaction.
- A small M means weak interaction.

Engineering example: wireless charging

In wireless charging, the transmitter coil and receiver coil are separated by an air gap. The coupling coefficient k is much smaller than in a transformer with an iron core.

If the receiver is misaligned, less flux from the transmitter passes through it. Then M decreases, the induced voltage decreases, and the transmitted power decreases.

Analogy: shared bed sheet



Imagine two people laying on a bed holding the same bed sheet at different positions.

- \(L_{1{\rm H}}\):
  how strongly winding 1 couples its current into the shared magnetic path,
  like person 1 moving the sheet at their hand.
- \(L_{2{\rm H}}\):
  how strongly winding 2 couples its current into the shared magnetic path,
  like person 2 moving the sheet at their hand.
- \(M\):
  how strongly the motion from one hand is felt at the other hand through the same sheet.
- \(L_{1\sigma}\), \(L_{2\sigma}\):
  local wrinkles near one hand. They move locally but do not effectively reach the other hand.

In transformer language, \(L_{1{\rm H}}\) and \(L_{2{\rm H}}\) describe each winding's connection to the shared main flux path.
The mutual inductance \(M\) describes the transfer between the two windings through this shared path.

voltages by mutual inductances and resistances

For positive coupling, we get the following complex representation (since $u(t) = L\frac{{\rm d}i}{{\rm d}t} \: \longrightarrow \:\underline{U}=j\omega L$):

\[\begin{align*} \underline{U}_1 &= R_1 \underline{I}_1 + {\color{green} {j\omega L_{1\rm H} \underline{I}_1 }} + {\color{blue}{j\omega M \underline{I}_2 }}, \\[4pt] \underline{U}_2 &= R_2 \underline{I}_2 + {\color{blue} {j\omega M \underline{I}_1 }} + {\color{green}{j\omega L_{2\rm H} \underline{I}_2 }}. \end{align*} \]
For negative coupling, the sign of the \(M\)-term changes in the chosen equation system, see figure 2 and figure 3.

Fig. 2: Positive coupling: currents enter corresponding dotted terminals.

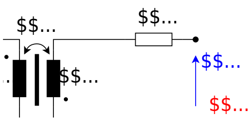
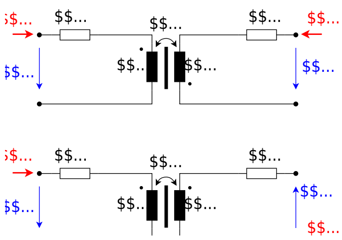


Fig. 3: Negative coupling: only one current enters a dotted terminal.



Tunnel Analogy for AC circuits

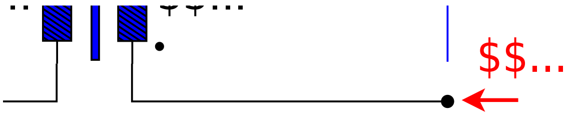
The dots are like matching openings for magnetic action.
A positive current (e.g. $\underline{I}_1$) entering the dotted terminal of one winding produces a positive induced voltage (e.g. aligned with $\underline{U}_2$) at the dotted terminal of the other winding.
With only a load R_2 connected to the secondary side, this voltage tends to drive current out of the dotted terminal into the load ($\underline{I}_2$ has to be inverted, since the transformer is a source then).

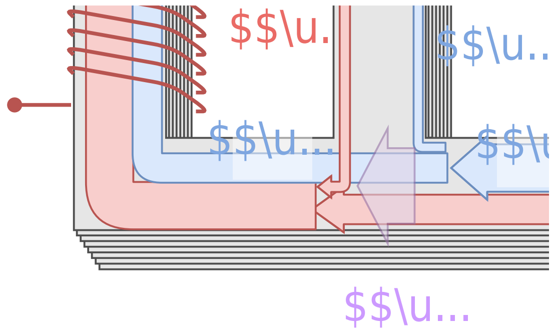
Real transformer: leakage and losses

In a real transformer, not all flux links both windings.

- The **main flux** $\Phi_{\rm H}$ links primary and secondary winding.
- The **primary leakage flux** $\Phi_{1\sigma}$ mainly links only the primary winding.
- The **secondary leakage flux** $\Phi_{2\sigma}$ mainly links only the secondary winding.

Fig. 5: Main flux and leakage fluxes in a real transformer.





The real flux linkage equations become

$$\begin{align*} \underline{\Psi}_1 &= \underbrace{{\color{blue}{L_{1{\rm H}}\underline{I}_1+M\underline{I}_2}}}_{\text{main magnetic path}} + \underbrace{{\color{orange}{L_{1\sigma}\underline{I}_1}}}_{\text{primary leakage}}, \\[4pt] \underline{\Psi}_2 &= \underbrace{{\color{blue}{L_{2{\rm H}}\underline{I}_2+M\underline{I}_1}}}_{\text{main magnetic path}} + \underbrace{{\color{orange}{L_{2\sigma}\underline{I}_2}}}_{\text{secondary leakage}}. \end{align*}$$

Equivalently,

$$\begin{align*} \underline{\Psi}_1 &= L_1\underline{I}_1+M\underline{I}_2, & L_1&=L_{1{\rm H}}+L_{1\sigma}, \\ \underline{\Psi}_2 &= L_2\underline{I}_2+M\underline{I}_1, & L_2&=L_{2{\rm H}}+L_{2\sigma}. \end{align*}$$

The winding resistances R_1 and R_2 cause copper losses:

$$\begin{align*} P_{\rm Cu,1}=R_1I_1^2, \qquad P_{\rm Cu,2}=R_2I_2^2. \end{align*}$$

$$\begin{align*} \underline{U}_1 &= \underbrace{{\color{red}{R_1\underline{I}_1}}}_{\text{primary copper drop}} + \underbrace{{\color{orange}{j\omega L_{1\sigma}\underline{I}_1}}}_{\text{primary leakage drop}} + \underbrace{{\color{blue}{j\omega L_{1{\rm H}}\underline{I}_1+j\omega M\underline{I}_2}}}_{\text{main magnetic coupling}}, \\[6pt] \underline{U}_2 &= \underbrace{{\color{red}{R_2\underline{I}_2}}}_{\text{secondary copper drop}} + \underbrace{{\color{orange}{j\omega L_{2\sigma}\underline{I}_2}}}_{\text{secondary leakage drop}} + \underbrace{{\color{blue}{j\omega L_{2{\rm H}}\underline{I}_2+j\omega M\underline{I}_1}}}_{\text{main magnetic coupling}}. \end{align*}$$

Color scheme for the equivalent equations

In the previous formulas:

- red terms:
  winding resistance and copper loss, which convert electrical energy into heat.
- orange terms:
  leakage flux, which is unwanted but unavoidable, due to the non-zero permeabilits of air.
- blue terms:
  useful main magnetic coupling, which is responsible for transformer action.

Analogy: useful road and side roads

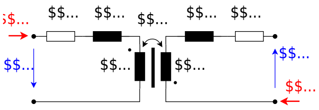
Think of the main flux as traffic on the useful road between two cities. Traffic on side roads still exists, but it does not help transport goods between the two cities.

- main flux: useful road between primary and secondary winding,
- leakage flux: side roads that return locally,
- winding resistance: friction that turns useful energy into heat.

Reduced equivalent circuit referred to the primary side

Since we know, that we can transform the current an voltage by the transformer, we can use this also to simplify the circuit.

Fig. 6: equivalent circuit of a real transformer



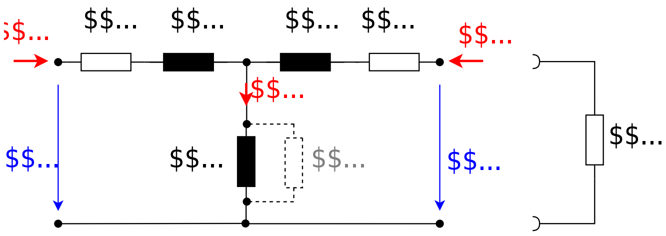
For calculations it is convenient to move all secondary-side quantities to the primary side.
This is called **referring** or **transforming** the secondary side to the primary side.

\[\begin{align*} \boxed{ \underline{U}'_2=n\underline{U}_2 } \qquad \boxed{ \underline{I}'_2=\frac{1}{n}\underline{I}_2 } \end{align*} \]

The secondary resistance and leakage reactance are transformed by \(n^2\) since $R = U / I$:

\[\begin{align*} \boxed{ R'_2=n^2R_2 } \qquad \boxed{ X'_{2\sigma}=n^2X_{2\sigma} } \end{align*} \]

Fig. 7: reduced equivalent circuit of a real transformer



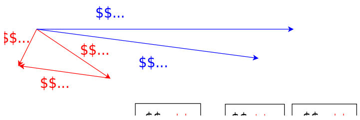
In the reduced equivalent circuit:

- R_1 and R'_2 model copper losses.
- $jX_{1\sigma}$ and $jX'_{2\sigma}$ model leakage flux.
- $jX_{1{\rm H}}$ models the magnetizing branch.
- $R_{\rm Fe}$ is placed parallel to $jX_{1{\rm H}}$ to model iron losses.
- $R_{\rm L}$ is a load resistor for the phasor diagram.

The phasor diagram is shown in figure 8.

- The upper one only shows the main voltages and currents.
  $\underline{U}'_2$ is parallel to $\underline{I}'_2$, since there is a load resistor on the right side.
  The transformer is a source, therefore $\underline{U}'_2$ is antiparallel to $\underline{I}'_2$.
- The lower one shows all voltages and currents.

Fig. 8: phasor diagram of a real transformer



Why the reduced circuit is useful

Once all quantities are referred to one side, the transformer can be calculated like an AC network with impedances.
This uses the same method as complex network calculation: replace components by impedances and apply Kirchhoff's laws.

No-load operation of the real transformer

No-load operation means that the secondary side is open:

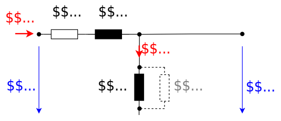
$$\begin{align*} \underline{I}_2=0. \end{align*}$$

The primary side still draws a small no-load current $\underline{I}_{10}$. This current has two parts:

$$\begin{align*} \underline{I}_{10} = \underline{I}_{\rm Fe} + \underline{I}_{\rm m}. \end{align*}$$

- $\underline{I}_{\rm Fe}$: current through $R_{\rm Fe}$, in phase with voltage, represents iron losses.
- $\underline{I}_{\rm m}$: magnetizing current through $jX_{1{\rm H}}$, approximately 90° lagging.

Fig. 9: No-load phasor diagram: the no-load current is the sum of iron-loss current and magnetizing current.



The technical voltage ratio is often defined from the no-load voltages. Here it is denoted by $\ddot{u}$:

$$\boxed{ \begin{align*} \ddot{u} = \frac{\text{higher voltage}}{\text{lower voltage}} \bigg|_{\rm no~load}. \end{align*} }$$

For a step-down transformer:

$$\begin{align*} \ddot{u} = \frac{U_{1{\rm N}}}{U_{20}}. \end{align*}$$

Here $U_{1{\rm N}}$ is the rated primary voltage and U_{20} is the open-circuit secondary voltage.

> Because of real voltage drops and magnetizing effects,
>
> $$\begin{align*} \ddot{u}\neq n, \end{align*}$$
>
> but for many practical transformers
>
> $$\begin{align*} \ddot{u}\approx n. \end{align*}$$

Short-circuit operation of the real transformer

In the short-circuit test, the secondary side is shorted:

$$\begin{align*} \underline{U}_2=0. \end{align*}$$

Because the required primary voltage is small, the magnetizing branch is often neglected:

$$\begin{align*} X_{1{\rm H}},\;R_{\rm Fe} \gg X'_{2\sigma},\;R'_2. \end{align*}$$

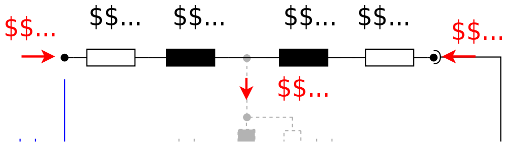
This gives the short-circuit equivalent circuit with

\[\begin{align*} \boxed{ R_{\rm k}=R_1+R'_2 } \qquad \boxed{ X_{\rm k}=X_{1\sigma}+X'_{2\sigma} } \end{align*} \]

and

\[\begin{align*} \underline{Z}_{\rm k} = R_{\rm k}+jX_{\rm k}. \end{align*} \]

Fig. 10: Short-circuit equivalent circuit of a real transformer.



Definition: rated short-circuit voltage

The **rated short-circuit voltage** \(U_{1{\rm k}}\) is the primary voltage that must be applied while the secondary side is shorted so that rated primary current \(I_{1{\rm N}}\) flows.

As a relative value:

\[\begin{align*} \boxed{ u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% } \end{align*} \]

Small \(u_{\rm k}\) means: small internal impedance and high possible fault current. Large \(u_{\rm k}\) means: stronger current limitation and larger voltage drop under load.

The continuous short-circuit current for rated primary voltage is

\[\begin{align*} \boxed{ I_{1{\rm k}} = \frac{U_{1{\rm N}}}{U_{1{\rm k}}}\cdot I_{1{\rm N}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} } \end{align*} \]

where $u_{\rm k}$ is inserted as a percentage value.

Common pitfalls

- …

Exercises

Worked examples

…

Embedded resources

Explanation (video): …